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Characterization of Sound Radiation by Unresolved Scales of Motion in Computational Aeroacoustics

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CHARACTERIZATION OF SOUND RADIATION BY UNRESOLVED SCALES OF MOTION IN COMPUTATIONAL AEROACOUSTICS*

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Abstract. Evaluation of the sound sources in a high Reynolds number turbulent flow requires time-accurate resolution of an extremely large number of scales of motion. Direct numerical simulations will therefore remain infeasible for the forseeable future; although current large eddy simulation methods can resolve the largest scales of motion accurately, they must leave some scales of motion unresolved. *A priori* studies show that acoustic power can be underestimated significantly if the contribution of these unresolved scales is simply neglected.

In this paper, the problem of evaluating the sound radiation properties of the unresolved, subgrid-scale motions is approached in the spirit of the simplest subgrid stress models: the unresolved velocity field is treated as isotropic turbulence with statistical descriptors evaluated from the resolved field. The theory of isotropic turbulence is applied to derive formulas for the total power and the power spectral density of the sound radiated by a filtered velocity field. These quantities are compared with the corresponding quantities for the unfiltered field for a range of filter widths and Reynolds numbers.

Key words. aeroacoustics, subgrid sound, large eddy simulation, sound sources

Subject classification. Physical Sciences

1. Introduction. The evaluation of the sound sources in high Reynolds number turbulent flows presents a fundamental problem for computational aeroacoustics. At one extreme, if a turbulence transport model is used to compute single-point single-time moments of the turbulence, the uncertainty of the turbulence model is compounded by the uncertainty of modeling the two-point two-time statistics required to evaluate the sound source.

At the other extreme, direct numerical simulation (DNS) requires that a large number of scales of motion be resolved accurately in time. Typical estimates [11] are that the resolution requirements are of the order of Re^3. Since the sound source depends on time correlations, in principle the entire flow history must be stored; this would impose prohibitive storage requirements even to compute the sound radiated by a model flow like isotropic turbulence. Further difficulties are posed if the acoustic field is to be resolved by DNS as well [11].

A practical compromise appears to be emerging in which the sound sources are computed by LES and are propagated to the far field by an acoustic analogy [10, 5] or by solution of the linearized Euler equations [4, 2].

If the sound source is computed by large eddy simulation (LES), then the acoustic calculation will evaluate the sound radiated by the resolved velocity field alone. It can be anticipated that this will result at least in the suppression of high-frequency sound. *A priori* studies [11, 14, 16] suggest that this type of numerical sound suppression can be significant and motivate the present theoretical study of the relationship between the sound radiated by the exact velocity field and the sound radiated by the filtered velocity field.

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This problem is addressed in the spirit of the simplest ideas used in subgrid stress modeling. Namely, we invoke Kolmogorov's theory of the universality of the small scales of motion in turbulence [3] and assume that the unresolved scales can be modeled as isotropic turbulence with statistical descriptors computed from the resolved velocity field. Using the theory of the space-time properties of isotropic turbulence, we can construct models of the exact and filtered velocity fields and compare the sound radiated by both fields as a function of filter width.

A different approach to subgrid-scale sound radiation is proposed by [14]. The present work differs from this primarily in the emphasis on time correlation modeling.

The present analysis is closely related to another modeling method in CAA, the *stochastic synthesis* of the subgrid motions [1, 4]. Like these methods, the present analysis depends on a model for the two-point two-time properties of the subgrid motions. Thus, our model (Eqs. (2.7) and (2.8) below), could be used as the statistical descriptor required to synthesize the subgrid sound sources.

Precise comparisons with existing *a priori* studies [11, 14, 16] are hampered by the very low Reynolds numbers of the direct simulations; the present analysis is appropriate when a Kolmogorov inertial range exists. Comparisons based on the non-universal spectra obtained in low Reynolds number simulations is possible, but less tractable analytically. This issue is discussed later.

2. The exact and filtered sound source. Denote the exact fluctuating velocity field by $\mathbf{u}(\mathbf{x}, t)$, and the filtered field by $\bar{\mathbf{u}}(\mathbf{x}, t)$. Only the sharp Fourier cutoff filter will be treated explicitly here, for which the space Fourier transform of the resolved velocity field is

$$\bar{\mathbf{u}}(\mathbf{k}, t) = \begin{cases} \mathbf{u}(\mathbf{k}, t) & \text{if } k \leq k_L \\ 0 & \text{if } k \geq k_L \end{cases} \tag{2.1}$$

where

$$k_L = \pi/\Delta \tag{2.2}$$

and Δ equals the physical space filter size. Extension of the analysis to other filters would be straightforward.

Lighthill's [10] formula for the acoustic pressure fluctuations in the far-field is

$$p(\mathbf{x}, t) = \frac{1}{4\pi c^2} \frac{x_i x_j}{x^3} \int_V d\mathbf{y} \; \ddot{T}_{ij}(\mathbf{y}, t - \frac{\mid \mathbf{x} - \mathbf{y} \mid}{c}) \tag{2.3}$$

where the form of the Lighthill tensor for sound radiation by subsonic flow

$$T_{ij}(\mathbf{y}, t) = \rho u_i(\mathbf{y}, t) u_j(\mathbf{y}, t) \tag{2.4}$$

is used. In Eqs. (2.3) and (2.4), V denotes the source region, ρ is the mean density, c is the speed of sound in the far-field, and the vector $\mathbf{x}$ connects the measurement point to some representative point in the source region.

The sound radiated by the filtered velocity field has the same form as Eq. (2.3), but with the Lighthill tensor computed from the resolved, rather than the exact velocity:

$$\bar{T}_{ij}(\mathbf{y}, t) = \rho \bar{u}_i(\mathbf{y}, t) \bar{u}_j(\mathbf{y}, t) \tag{2.5}$$

Kraichnan [7] gave the far-field acoustic power spectral density as

$$p(\omega) = \pi(\omega^4/2c^8)\langle \mid n_i n_j T_{ij}(\omega \mathbf{n}/c, \omega) \mid^2 \rangle \tag{2.6}$$

where $\mathbf{n}$ is the unit vector in the direction of $\mathbf{x}$, ω is the frequency of the radiated sound, and $T_{ij}(\mathbf{k},\omega)$ is the space-time Fourier transform of the fluctuating quantity $T_{ij}(\mathbf{x},t)$. $T_{ij}(\mathbf{k},\omega)$ can be written in terms of the space-time Fourier transform of the velocity field as

$$T_{ij}(\mathbf{k},\omega) = \rho \int \delta(\mathbf{k}-\mathbf{p}-\mathbf{q}) d\mathbf{p} d\mathbf{q} \int_{\infty}^{\infty} d\omega' \; u_i(\mathbf{p},\omega-\omega') u_j(\mathbf{q},\omega') \tag{2.7}$$

It is necessary to express the result of Eq. (2.6) in terms of statistics of the velocity field. To this end, replace T_{ij} in Eq. (2.6) by result in Eq. (2.7). Close the resulting fourth-order velocity moment by quasinormality, and use the correlation function for isotropic turbulence

$$Q_{ij}(\mathbf{k},\omega) = \frac{1}{4\pi k^2} E(k) R(k,\omega) [\delta_{ij} - k_i k_j k^{-2}] \tag{2.8}$$

where $E(k)$ is the energy spectrum and $R(k,\omega)$ is the time correlation function.

It can be shown [13] that Eq. (2.6) implies that sound is radiated only by interactions between incompressible modes nearly of the type $u_i(\mathbf{k},t)$ and $u_j(-\mathbf{k},t)$. This approximation treats the sound waves as infinitely long; equivalently, it ignores the so-called *retarded time effect*. Introducing this approximation, the result [12] is

$$p(\omega) = C \frac{\omega^4}{Vc^5} \int_0^{\infty} dk \; E(k)^2 k^{-3} \hat{R}(k,\omega) \tag{2.9}$$

where $\hat{R}$ denotes the frequency convolution

$$\hat{R}(k,\omega) = \int_{-\infty}^{\infty} d\omega' \; R(k,\omega-\omega') R(k,\omega') \tag{2.10}$$

3. Sound radiation by the filtered velocity field. To complete the calculation of the far-field acoustic power spectral density function using Eq. (2.9), models for the energy spectrum $E(k)$ and the time correlation function $R(k,\omega)$ are needed. Since these will describe the unresolved motions, it is necessary to assume that these motions can be described reasonably well by some universal properties; for this description, we will apply the Kolmogorov theory of the inertial range [3].

Our previous work [12] discusses the issue of time correlations in detail. It is concluded that sound radiation is determined by *Eulerian* time correlations, for which turbulence theory [9, 15, 6] demonstrates the similarity form

$$R(k,\omega) = R(\frac{\omega}{Vk}) \tag{3.1}$$

for inertial range scales, where V is the rms of the fluctuating velocity. A similar formula is proposed by Bailly and Juvé [2], although this work treats the frequencies corresponding to each wavevector as random variables.

Replacing $\hat{R}$ by the similarity form corresponding to Eq. (3.1),

$$p(\omega) = C \frac{\omega^4}{Vc^5} \int_0^{\infty} dk \; E(k)^2 k^{-3} \hat{R}(\frac{\omega}{Vk}) \tag{3.2}$$

Next, let $E(k)$ be the Kolmogorov spectrum

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3} \tag{3.3}$$

In reality, this form only applies to a finite range of scales $k_0 < k < k_d$, where k_0 is the inverse integral scale, and k_d is proportional to the inverse Kolmogorov scale. But the subsequent calculation will show that the exact form of the spectrum in the region of large scales $k \leq k_0$ is not important for the calculation of subgrid sound. Similarly, the spectrum in the dissipation range with $k \geq k_d$ can be neglected because of its insignificant energy content. Substituting Eq. (3.3) in Eq. (3.2),

$$p(\omega) = C\frac{\varepsilon^{4/3}}{c^5}\omega^4 \int_0^\infty dk\ k^{-19/3}\hat{R}(\frac{\omega}{Vk}) \tag{3.4}$$

If the time correlation function $\hat{R}$ decays sufficiently rapidly at ∞, this integral is finite for k near 0; consequently, as noted earlier, the precise form of $E(k)$ for small k is not needed for this calculation. Regardless of the functional form of $\hat{R}$, power counting shows that for large ω,

$$p(\omega) \approx C\frac{V^{13/3}\varepsilon^{4/3}}{c^5}\omega^{-4/3} \tag{3.5}$$

Next, consider the sound radiated by the filtered velocity field. Denote its power spectral density by $p_L(\omega)$ to indicate the dependence on the filter scale k_L. We noted in the previous section that ignoring the retarded time effect means that only mode pairs of the form $u_i(\pm\mathbf{k}, t)$ interact to radiate sound [13]. If so, the sound radiated by the filtered velocity field $\bar{u}_i(\mathbf{k}, t)$ is found from Eq. (3.4) by restricting the k integration to the resolved scales:

$$p_L(\omega) = C\frac{\omega^4}{Vc^5}\int_0^{k_L} dk\ E(k)^2 k^{-3}\hat{R}(\frac{\omega}{Vk}) \tag{3.6}$$

with the obvious analog of Eq. (3.6) for the Kolmogorov spectrum

$$p_L(\omega) = C\frac{\varepsilon^{4/3}}{c^5}\omega^4 \int_0^{k_L} dk\ k^{-19/3}\hat{R}(\frac{\omega}{Vk}) \tag{3.7}$$

Analytical results will require assuming a specific functional form for the time correlation function. We note three forms:

1. Kraichnan's [8] result

$$R(x) = \frac{J_1(x)}{x} \tag{3.8}$$

2. the Markovian approximation

$$R(x) = \exp(- \mid x \mid) \tag{3.9}$$

3. the Gaussian approximation

$$R(x) = \exp(-x^2) \tag{3.10}$$

where x is the similarity variable

$$x = \alpha\frac{\omega}{Vk} \tag{3.11}$$

The constant α should be chosen to match the second order Taylor coefficient of Eqs. (3.8)–(3.11) to the short-time expansion of the Navier-Stokes equations following the analysis of Kaneda [6].

The analytically most convenient form is the Gaussian of Eq. (3.10), which is tentatively adopted here in order to illustrate the derivation of a theory of subgrid-scale sound. Regardless of which of Eqs. (3.8)-(3.10) is used, the final result can be written in the form

$$\bar{p}_L(\omega) = C\frac{\varepsilon^{4/3}V^{13/3}}{c^5}\omega^{-4/3}F(\frac{\omega}{Vk_L}) \tag{3.12}$$

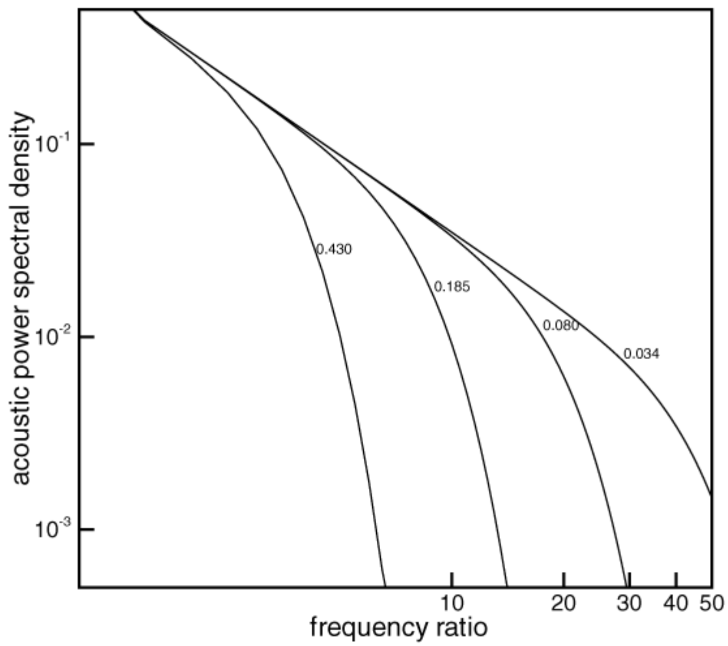

FIG. 3.1. *Effect of filter size on acoustic power spectral density.*

For Eq. (3.10), the function F can be expressed as an incomplete gamma function.

The effect of filter width on the spectrum of sound radiated by a filtered velocity field is shown in Figure 3.1 in which the dimensionless power spectrum is plotted as a function of normalized frequency ω/ω_0 where ω_0 is the frequency integral scale. The filter width is converted to a frequency ω_L through the relation

$$\omega_L = V k_L \tag{3.13}$$

and the spectrum in Eq. (3.12) is plotted for the values $\omega_0/\omega_L = k_0/k_L = .9^4 \approx 0.43, .9^8 \approx 0.18, .9^{16} \approx 0.08, .9^{32} \approx 0.03$ ranging from extremely coarse to very fine resolution of the fluctuating field. The most conspicuous effect of coarsening the resolution is suppression of high frequency sound, although there is some effect at all frequencies.

In Figure 3.1, it is assumed that the inertial range extends over all scales and consequently that sound is radiated at all frequencies. But if it is assumed instead that the dissipation range spectrum is simply zero, then the acoustic spectrum would have the form of Eq. (3.12) with the filter scale k_L replaced by the dissipation scale k_d. In this case, comparison between two curves in Figure 3.1 shows the effect of filtering a velocity field with a finite k_d.

Perhaps more immediately important is the effect of filter width on the computed total acoustic power. The exact total acoustic power is

$$P = \int_0^\infty d\omega \; p(\omega) \tag{3.14}$$

and the total power radiated by the resolved scales is

$$P_L = \int_0^\infty d\omega \; p_L(\omega) \tag{3.15}$$

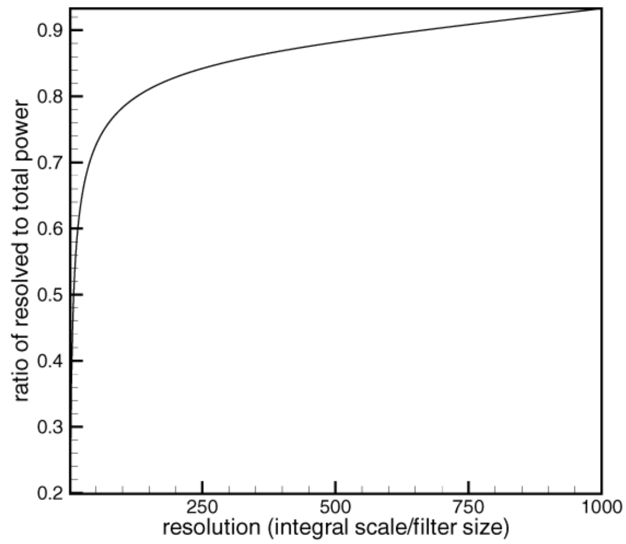

FIG. 3.2. *Effect of filtering on total power.*

Using Eq. (3.12), the ratio of resolved to total acoustic power is

$$\frac{P_L}{P} = \frac{1}{3F(0)} \int_1^{\omega_L/\omega_0} d\tilde{\omega}(\tilde{\omega})^{-4/3} F(\tilde{\omega}\frac{\omega_0}{\omega_L}) \tag{3.16}$$

This function is shown in Figure 3.2 as a function of the variable $\omega_L/\omega_0 = k_L/k_0$, the ratio of the integral scale to the filter size. Since the inertial range extends over all scales, even the largest value shown, which corresponds to a filter width of 0.001 times the integral scale, is only within about 1 dB of the total power. For filter widths less than 0.01 times the integral scale, the resolved power rises rapidly: at 0.01 times the integral scale, the resolved power is within about 3 dB of the total.

As before, the ratio P_L/P_d where P_d is the power in a Kolmogorov spectrum cut off sharply at scale k_d, is a good approximation for the ratio of resolved to total power for a finite Reynolds number flow. Figure 3.2 therefore provides a single curve showing the combined effects of Reynolds number and resolution.

By setting $k_d/k_0 = 100.$, we obtain a Reynolds number of about 460. The ratio of resolved to theoretical total power P_L/P_d is plotted for this case in Figure 3.3, together with the same ratio in the infinite Reynolds number limit. The "resolution" is the ratio of the integral scale to the filter size: although it might be more natural to define resolution as the ratio of the filter size to the Kolmogorov scale, the results for different Reynolds numbers could not be compared directly since the Kolmogorov scales corresponding to the two different Reynolds numbers would be unequal. As expected, the unresolved power in the infinite Reynolds number limit always exceeds the unresolved power in the finite Reynolds number flow. In both cases, the subgrid power drops sharply as the resolution is increased over low values, but tends to drop slowly at higher resolution.

All of these estimates of the acoustic power in the subgrid scales must be considered preliminary because of the assumption of the Gaussian time correlation function which has been chosen largely for analytical convenience.

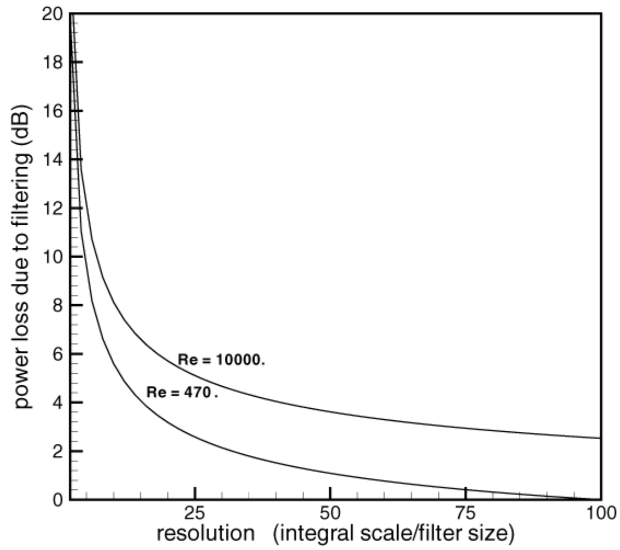

FIG. 3.3. *Power loss due to filtering: Effect of Reynolds number.*

The problem of acoustic power radiated by subgrid scales has been addressed in *a priori* studies by Piomelli *et al.* [11] for the model problem of channel flow and by Witkowska *et al.* [16] for the problem considered here of isotropic turbulence. The latter study considers both forced, steady turbulence and the more commonly studied problem of decaying turbulence. Comparison with these results requires some care because of the limited Reynolds numbers of the simulations. This comparison must be based on results like Eq. (3.6) expressed in terms of a numerically given spectrum $E(k)$ instead of the Kolmogorov spectrum used here.

4. Conclusions. The present work is a first step to an analytical theory of the subgrid contribution to radiated sound. It shows how the theory of isotropic turbulence can be applied to derive a theory of subgrid-scale sound radiation. Refinement of this model will require closer investigation of the time correlation function, which is the key ingredient of our analysis.

The close connection of this work to the method of stochastic synthesis advanced by Bailly *et al.* [1, 2] and by Béchara *et al.* [4] was noted earlier. Although this application has not been developed explicitly here, the present theory is based on a two-point two-time model of the subgrid scales which could also be used to synthesize the subgrid-scale motions.

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