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EFFECTS OF EDDY VISCOSITY ON TIME CORRELATIONS IN LARGE EDDY SIMULATION

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Abstract. Subgrid-scale (SGS) models for large eddy simulation (LES) have generally been evaluated by their ability to predict single-time statistics of turbulent flows such as kinetic energy and Reynolds stresses. Recent applications of large eddy simulation to the evaluation of sound sources in turbulent flows, a problem in which time correlations determine the frequency distribution of acoustic radiation, suggest that subgrid models should also be evaluated by their ability to predict time correlations in turbulent flows.

This paper compares the two-point, two-time Eulerian velocity correlation evaluated from direct numerical simulation (DNS) with that evaluated from LES, using a spectral eddy viscosity, for isotropic homogeneous turbulence. It is found that the LES fields are too coherent, in the sense that their time correlations decay more slowly than the corresponding time correlations in the DNS fields. This observation is confirmed by theoretical estimates of time correlations using the Taylor expansion technique. The reason for the slower decay is that the eddy viscosity does not include the random backscatter, which decorrelates fluid motion at large scales. An effective eddy viscosity associated with time correlations is formulated, to which the eddy viscosity associated with energy transfer is a leading order approximation.

Key words. eddy viscosity, time correlation, large eddy simulation

Subject classification. Fluid Mechanics

1. Introduction. In large eddy simulation of turbulent flows, large-scale velocity fields are computed directly from the filtered Navier-Stokes (N-S) equation, while the effects of small-scale velocity fields on large-scale velocity fields are modeled using a SGS model. The SGS models are constructed to represent energy drain from large scales to small scales, and local inverse energy transfer, i.e., energy backscatter (from small scales to large scales). It is desirable that the SGS models can produce a LES field which correctly predicts the large-scale statistics of the N-S field. A direct evaluation [1, 2, 3] of the SGS models is to compare statistics of the LES fields with those of the filtered N-S fields. There has been extensive work comparing single-time statistics of the LES and DNS fields. For recent reviews, see [4, 5]. However, there is little work concerning two-time statistics. This paper investigates the effects of the eddy viscosity SGS models on the time correlations of the LES fields. The research is motivated by use of LES in the aeroacoustics [4, 6], where sound radiation is dependent on Eulerian time correlations.

Time correlations are among the simplest statistical properties of turbulent flow. We will consider Eulerian time correlations in isotropic homogeneous turbulence, whose Fourier transformation is expressed as

$$C(k,\tau) = \langle \mathbf{u}(\mathbf{k}, t+\tau) \cdot \mathbf{u}(\mathbf{k}, t) \rangle. \tag{1.1}$$

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where $\mathbf{u}(\mathbf{k}, t)$ is a Fourier mode of the velocity field at the wavenumber vector $\mathbf{k}$ with $k = |\mathbf{k}|$. The bracket "$\langle\ \rangle$" means ensemble averaging. The Eulerian time correlation measures the temporal changes seen by an observer in a fixed reference frame with zero mean velocity. For a possible equilibrium range at high frequencies, such an observer sees small scale eddies being swept past him by larger scale eddies. The correlation coefficient [8, 9] between the LES and DNS fields has been used to evaluate the ability of LES to predict flow evolution. It was demonstrated that the correlation coefficient will decay to zero after a few eddy turnover times. This implies the LES and DNS fields will be completely decorrelated. However, this result is not discouraging. In fact, two DNS fields which are initially identical at large scales but slight different at small scales will also become completely decorrelated, although their uncoupled statistics still remain equal. Even independent random fields can be statistically identical. Therefore, the relevant question to ask is if the LES fields can reproduce the statistics of the DNS fields, whether the LES and DNS fields are correlated or not. In this paper, the validity of the LES is evaluated by its ability to reproduce the Eulerian time correlations of the DNS field.

Current SGS models can be categorized into several groups: eddy viscosity, stochastic, similarity, assumed SGS velocity models [5] and multiscale model [10]. The most extensively used SGS model is based on the eddy-viscosity assumption: the effects of subgrid scale motion is supposed to be similar to that of molecular dissipation, but with an eddy viscosity which depends on both wavenumber and the cutoff. Many eddy viscosity models have been developed including the Smagorinsky model, the kinetic-energy model and the dynamical model. In isotropic homogeneous turbulence, the eddy viscosity can be formulated in terms of the energy balance equation:

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) = T(k,t), \tag{1.2}$$

where $E(k,t)$ is the kinetic energy spectrum, ν the kinematic viscosity and $T(k,t)$ the energy transfer. One introduces a cutoff wavenumber k_c such that only scales larger than $1/k_c$ will be resolved explicitly. The energy transfer can be separated into two parts

$$T(k,t) = T^{<}(k|k_c,t) + T^{>}(k|k_c,t), \tag{1.3}$$

where $T^{<}(k|k_c,t)$ is the part of the total energy transfer induced by triad interactions among waves $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{k}$ satisfying $\mathbf{p}+\mathbf{q}=\mathbf{k}$ such that p and q are less than k_c, and $T^{>}(k|k_c,t)$ the part of the total energy transfer induced by triad interaction among the wavenumber $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{k}$ satisfying $\mathbf{p}+\mathbf{q}=\mathbf{k}$ such that p or q is larger than k_c. Hereinafter, the superscript $<$ implies the part of any quantity involving only wavenumbers less than the cutoff k_c, while the superscript $>$ denotes the remainder, in which at least one of the wavenumbers is larger than the cutoff k_c. The eddy viscosity is defined by [11]:

$$\nu_t(k|k_c,t) = -\frac{T^{>}(k|k_c,t)}{2k^2 E(k,t)}, \tag{1.4}$$

so that finally

$$\left[\frac{\partial}{\partial t} + 2(\nu+\nu_t)k^2\right] E(k,t) = T^{<}(k|k_c,t). \tag{1.5}$$

In the formulation of the papers [12, 13]

$$\nu_t(k|k_c,t) = \nu_t^+(k|k_c)\sqrt{\frac{E(k_c,t)}{k_c}}, \tag{1.6}$$

where $E(k_c,t)$ is the energy spectrum at the cutoff wavenumber k_c and $\nu_t^+(k|k_c)$ is

$$\nu_t^+(k|k_c) = 0.267 + 9.21\exp(-3.03k_c/k). \tag{1.7}$$

This eddy viscosity model has been extensively studied by DNS and experiments. These studies show that it does model the energy dissipation at small scales but misses the energy backscatter from small scales to large scales. We will show that the lack of the random backscatter causes more coherence, in the sense that the time correlations of the LES fields decay more slowly than those of the DNS fields. We will focus on the eddy viscosity model (1.4). The results may be extended to other eddy-viscosity SGS models. In Section 2, we calculate the Eulerian time correlations for the DNS and LES fields, respectively. We will show that the LES fields predict larger time correlations. In Section 3, the reasons for the larger predictions are explained. We use the Taylor expansion technique to estimate the time correlations respectively for the N-S and LES fields. The effective eddy viscosity for time correlation is derived. Discussion and conclusions are given in Section 4.

2. Numerical calculation of time correlations. We carry out DNS and LES of isotropic homogeneous turbulence. The three-dimensional N-S equations are solved numerically in a periodic box of side 2π by the standard pseudospectral algorithm. In DNS, the flow cube is discretized uniformly into $N = 128$ grid points, which defines the wavenumber components in Fourier space as $k_j = \pm 0, 1, \cdots, N/2-1$ for $j = 1, 2, 3$. The aliasing errors are removed by the two-thirds truncation method. Consequently, the effective wavenumber space is by $|k| < N/3$. The initial condition is set to be isotropic and homogeneous with the energy spectrum $E(k,0) \propto k^4/k_0^5 \exp(-2(k/k_0)^2)$, where k_0 is the wavenumber at which the maximum of the energy spectrum occurs. By steady random forcing on the first two wavenumber shell $k < 2$, stationary conditions are achieved after some passage of time. In this study, the relevant parameters are: viscosity $\nu = 0.002$, time step size $dt = 0.005$, Taylor microscale wave number $R_\lambda = 40$, CFL number 0.46 and velocity derivative skewness -0.34. In LES, $N = 64$. We use the eddy viscosity model (1.6) and the sharp cutoff filter. All other parameters in the LES are kept to be the same as the DNS. The spectral codes for the DNS and LES are developed and implemented on the ICASE Beowulf 96 CPU cluster computer, Coral.

In Fig. 2.1, we plot the time correlations of the LES fields against the time delay. Fig. 2.1 clearly demonstrates the spatial scale dependence of the time correlations: the larger wavenumbers remain coherent for a longer time while the smaller wavenumbers are rapidly decorrelated. Fig. 2.2 and Fig. 2.3 show the same data against time delay normalized by eddy turnover time $\tau_e = [k\sqrt{kE(k)}]^{-1}$ and sweeping time $\tau_s = (uk)^{-1}$ respectively, where u is the rms velocity. The sweeping time is more suitable for collapsing the time correlation.

Fig. 2.4 shows the time correlations of the DNS and LES fields for different wavenumbers, $k = 5, 10, 15, 20$, spanning a range from the scale on the order of the integral scale to the scale on the upper end of resolved scale range. The times are normalized by the sweeping time at the largest wavenumber $k = 1$. It is evident that the time correlations of the LES fields decay more slowly than those of the DNS fields.

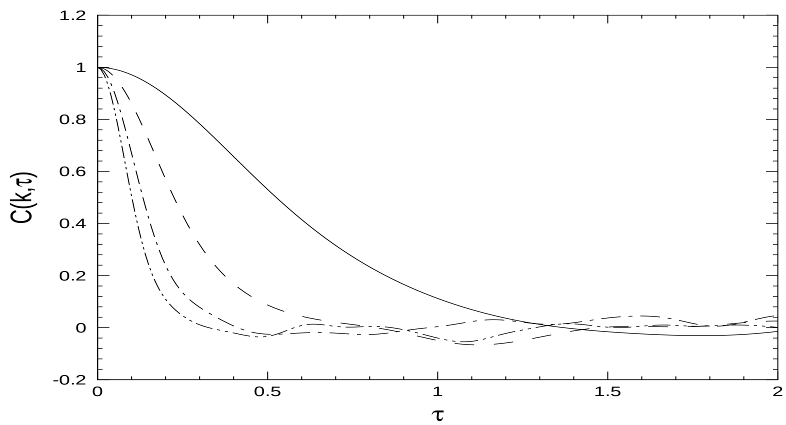

FIG. 2.1. *Time correlation $C(k,\tau)$ of the LES field as a function of time lag τ: $k = 5$ (solid), 10 (dash), 15 (dash-dot), 20 (dash-dot-dot).*

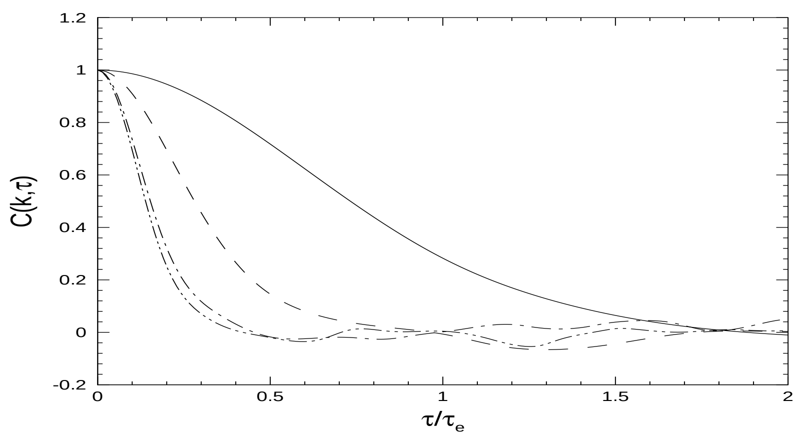

FIG. 2.2. *Time correlation $C(k,\tau)$ versus the time lag τ normalized by the local eddy-turnover time τ_e. Other details are as in Fig. 2.1.*

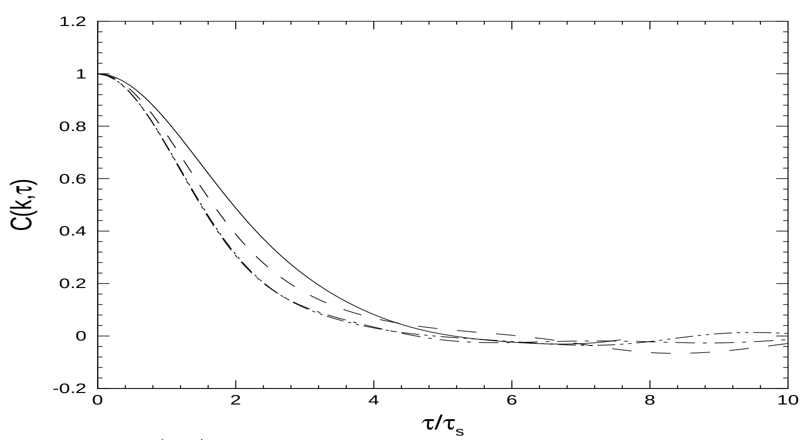

FIG. 2.3. *Time correlation $C(k,\tau)$ versus the time lag τ normalized by the sweeping time τ_s. Other details are as in Fig. 2.1.*

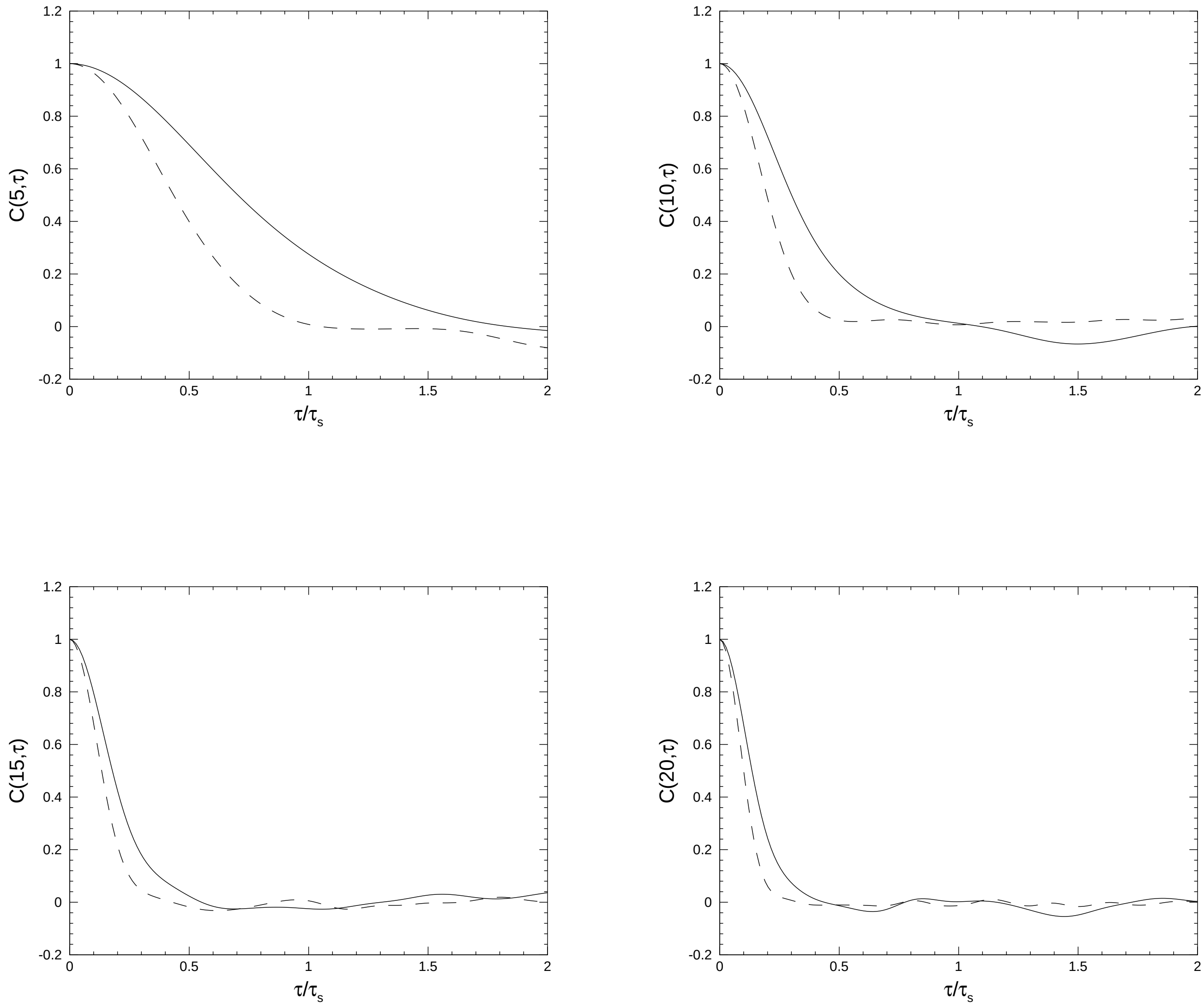

FIG. 2.4. *Time correlation $C(k,\tau)$ versus the time lag τ/τ_s. The solid line for the LES field and the dash line for the DNS field: $k = 5, 10, 15, 20$.*

3. The effects of eddy viscosity on time correlation. The numerical observations in the last section can be supported by theoretical estimates of time correlations. We will use the Taylor expansion technique to estimate time correlations for both the N-S and LES velocity fields. The Taylor expansions have been developed to construct approximations to statistics in turbulence. Kaneda's group [14, 15] has done extensive work on the Taylor expansion approach. Its central idea is to expand the time correlation into a power series of time lag τ. The coefficients in the Taylor series are determined by the governing equations.

The governing equations for the N-S fields u_α, $\alpha = 1, 2, 3$, in spectrum space are

$$\left(\frac{\partial}{\partial t} + \nu k^2\right) u_\alpha(\mathbf{k}, t) = M_{\alpha\beta\gamma}(\mathbf{p} + \mathbf{q} = \mathbf{k}) u_\beta(\mathbf{p}) u_\gamma(\mathbf{q}), \tag{3.1}$$

where

$$\begin{aligned} M_{\alpha\beta\gamma}(\mathbf{p} + \mathbf{q} = \mathbf{k}) &= M_{\alpha\beta\gamma}(\mathbf{k}) \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}}, \\ M_{\alpha\beta\gamma}(\mathbf{k}) &= \frac{1}{2i}\left(k_\beta P_{\alpha\gamma}(\mathbf{k}) + k_\gamma P_{\alpha\beta}(\mathbf{k})\right), \\ P_{\alpha\beta}(\mathbf{k}) &= \delta_{\alpha\beta} - k_\alpha k_\beta / k^2, \end{aligned} \tag{3.2}$$

and $\sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}}$ denotes a summation for all $\mathbf{p}$ and $\mathbf{q}$ such that $\mathbf{p} + \mathbf{q} = \mathbf{k}$.

The governing equations for the LES fields u_α, $\alpha = 1, 2, 3$, in spectrum space are

$$\left(\frac{\partial}{\partial t} + [\nu + \nu_t(k|k_c)]k^2\right) u_\alpha(\mathbf{k}, t) = M^<_{\alpha\beta\gamma}(\mathbf{p} + \mathbf{q} = \mathbf{k}) u_\beta(\mathbf{p}) u_\gamma(\mathbf{q}). \tag{3.3}$$

To facilitate comparison, we simply assume that the N-S and LES velocity fields $u_\alpha(\mathbf{k}, t)$ are identical at the time t. The velocity fields $u_\alpha(\mathbf{k}, t + \tau)$ at the later time $t + \tau$ are obtained from the equation.

The Taylor expansion of the time correlation (1.1) has the following form

$$\begin{aligned} C(k, \tau) &= C_0(k) + C_1(k)\tau + C_2(k)\tau^2/2 + \cdots, \\ C_n(k) &= \langle \frac{d^n u(\mathbf{k}, t)}{dt^n} u(-\mathbf{k}, t) \rangle. \end{aligned} \tag{3.4}$$

The time derivatives of the velocity fields can be formally calculated from the N-S equation. For a negligible viscosity ν, we obtain

$$\begin{aligned} C_0(k) &= \langle u_\alpha(\mathbf{k}, t) u_\alpha(-\mathbf{k}, t) \rangle, \\ C_1(k) &= \frac{1}{2}\left(T^< + T^>\right), \\ C_2(k) &= 2M_{\alpha\beta\gamma}(\mathbf{p} + \mathbf{q} = \mathbf{k}) M_{\beta\mu\sigma}(\mathbf{m} + \mathbf{n} = \mathbf{p}) \langle u_\mu(\mathbf{m}, t) u_\sigma(\mathbf{n}, t) u_\gamma(\mathbf{q}, t) u_\alpha(-\mathbf{k}, t) \rangle. \end{aligned} \tag{3.5}$$

Similarly, the Taylor coefficients of time correlations in the LES velocity fields can be obtained

$$\begin{aligned} \tilde{C}_0(k) &= \langle u_\alpha(\mathbf{k}, t) u_\alpha(-\mathbf{k}, t) \rangle, \\ \tilde{C}_1(k) &= \frac{1}{2}\left(T^< - 2\nu_t(k|k_c)k^2 \langle u_\alpha(\mathbf{k}, t) u_\alpha(-\mathbf{k}, t) \rangle\right), \\ \tilde{C}_2(k) &= 2M^< - M_{\alpha\beta\gamma}(\mathbf{p} + \mathbf{q} = \mathbf{k}) \nu_t(k|k_c)(p^2 + q^2 = k^2) \langle u_\beta(\mathbf{p}, t) u_\gamma(\mathbf{q}, t) u_\alpha(-\mathbf{k}, t) \rangle \\ &\quad + [\nu_t(k|k_c)k^2]^2 \langle u_\alpha(\mathbf{k}, t) u_\alpha(-\mathbf{k}, t) \rangle. \end{aligned} \tag{3.6}$$

Comparing the equations (3.5) with the equations (3.6), we find: (1) $C_0(k) = \tilde{C}_0(k)$; and (2) the eddy viscosity (1.4) implies $C_1(k) = \tilde{C}_1(k)$. In other words, the eddy viscosity (1.4) can be obtained solving the equation $C_1(k) = \tilde{C}_1(k)$. Recall that $C_1(k) = 0$; (3) $C_2(k) \neq \tilde{C}_2(k)$ for the eddy viscosity (1.4). This implies that the time microscale of the LES field is not equal to that in the DNS field. Therefore, the time correlations of the LES fields are different from those of the N-S fields. The numerical calculation indicates that the former is larger than the latter.

The results from numerical calculation and theoretical estimations can be understood by the following physical arguments: The contributions of small scales to large scales can be described as energy dissipation and random backscatter. The eddy viscosity correctly models the drain of energy from large scales to small scales but misses the random backscatter from small scales to large scales. This leads to a more coherent LES field. Therefore, the LES field evolves in a more correlated fashion, in the sense that their time correlations decay more slowly.

The eddy viscosity is introduced as a surrogate for energy drain from resolved scales to the subgrid scales. Its expression (1.4) is then found from the energy balance equation. Following this idea, the effective eddy viscosity for time correlation can be found from its governing equation as a surrogate of the contribution from subgrid scales to resolved scales.

The governing equation for time correlation may be written as

$$\begin{aligned} &\left(\frac{\partial}{\partial \tau} + \nu k^2\right) 2\langle u_\alpha(\mathbf{k}, t+\tau) u_\alpha(-\mathbf{k}, t)\rangle \\ &= M_{\alpha\beta\gamma}(\mathbf{p}+\mathbf{q}=\mathbf{k}) \left(\langle u_\beta(\mathbf{p}, t+\tau) u_\gamma(\mathbf{q}, t+\tau), u_\alpha(-\mathbf{k}, t)\rangle - c.c\right) \end{aligned} \tag{3.7}$$

where "$c.c$" represents the complex conjugate of the triple moment. We separate the nonlinear interaction on the right hand of the equation (3.7) into two parts: $M^{<}_{\alpha\beta\gamma}$ for p and $q < k$ and $M^{>}_{\alpha\beta\gamma}$ for p or $q \leq k$ such that $M_{\alpha\beta\gamma} = M^{<}_{\alpha\beta\gamma} + M^{>}_{\alpha\beta\gamma}$. Then, the effective eddy viscosity which describes the effects of the subgrid scales on the time correlations of the LES fields may be defined by

$$\nu_\tau(k|k_c) = \frac{M^{>}_{\alpha\beta\gamma}}{2k^2\langle u_\alpha(\mathbf{k}, t+\tau) u_\alpha(-\mathbf{k}, t)\rangle}. \tag{3.8}$$

We can also derive a Taylor expansion of the effective eddy viscosity $\nu_\tau(k|k_c)$ in power of time lag τ, leading to

$$\nu_\tau(k|k_c) = \nu_t(k|k_c) + O(\tau). \tag{3.9}$$

It indicates that the *normal* eddy viscosity (1.4) is the leading order approximation to the *effective* eddy viscosity (3.8). For small time lag, the normal eddy viscosity produces a satisfactory approximation. However, for large time lag, the corrections from higher order terms must be taken into account.

4. Discussion and conclusion. The normal eddy viscosity models the energy dissipation but ignores the random backscatter. It produces a more coherent LES field. It is shown by numerical calculation and theoretical estimation that the time correlations for the LES field decay more slower than those for the N-S field. The differences in time correlations between the LES and DNS fields decrease with increasing wavenumber. The effects of the eddy viscosity on time correlation have to be considered while applying LES to sound radiation.

We have shown that the normal eddy viscosity for the energy balance equation is a leading order approximation to the effective eddy viscosity for the time correlation equation. Therefore, it is difficult to construct a new surrogate of eddy viscosity, which could exactly satisfy both energy balance equation and time correlation equation. An effective eddy viscosity that could simultaneously produce a good approximation to the terms involving small scales in the energy balance and time correlation equations will be helpful. Noting that the information given by the eddy viscosity are only statistically averaged one, we need to account for the stochastical effects on the particular realization of the LES fields. The stochastical effects could be introduced by random forcing. Therefore, the decorrelation of the LES field may be resorted by including random force.

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