

# Analytical Predictions of Crack Growth Resistance In Notched Composite Panels

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## Introduction

Fibrous composite materials exhibit R-curve type behavior similar to the fracture behavior of metals [1]. R-curve type behavior in fiber-reinforced composites and in metals is a manifestation of existing toughening mechanisms. Damage tolerance studies have discovered similarities between metals and fiber-reinforced composites as well as some obvious differences. Unlike homogeneous metals, composite materials are made of multiple constituents. Therefore, the modes and types of failure are more complex. While the matrix material in the composite may have similarities to metal in terms of modes of crack opening, composites have fiber fractures and delaminations which affect the redistribution of load. This leads to the conclusion that the toughening mechanisms are different in composites and metals. Toughening mechanisms are physical phenomena responsible for crack growth resistance. Crack-tip plasticity, for example, is a dominant toughening mechanism in metals. The toughening mechanisms in composites are due to micro-cracking. Even though the toughening mechanisms are fundamentally different for metals and composites, the result for both materials is the elimination of the stress singularity and the effect on load redistribution. Because the notch-tip damage in a composite laminate is not a well defined and quantifiable crack like the notch-tip damage in metals, the terms crack growth and crack growth resistance may not be suitable for composites. The terms damage growth and damage growth resistance will be used when discussing composite laminates.

Only recently has the damage growth resistance of fiber-reinforced polymer matrix composites been investigated. A thorough study of the fracture toughness and residual strength of various fibrous composites was done by Poe [2]. Poe found in his investigations of brittle laminated composites that linear elastic fracture mechanics (LEFM) could be used to determine the fracture toughness of a notched composite panel. However, Poe et al. [1] and Coats [3] showed that LEFM does not provide accurate results for all notched composite panels, especially panels that exhibit higher levels of toughening. R-curve predictions for these notched composite panels were made reasonably well using a progressive damage methodology [3]. This methodology consists of the Allen-Harris non-linear damage dependent constitutive model [4-7]. The following sections will discuss the experimental and analytical investigations followed by R-curve predictions. These predictions were made using the progressive damage methodology.

## Experimental Procedure

The investigation of damage growth resistance in composite laminates is facilitated by the center-crack tension specimen, Figure 1. The panel in Figure 1 was manufactured for three panel widths, 10cm, 30cm, and 91cm, and three

notch lengths 2.54cm, 7.62cm, and 22.86cm using the material AS4/938 graphite fibers and epoxy matrix. The

laminates stacking sequence is  $[\bar{+}45/0/90/\bar{+}30/0]_S$  for all panels. The 10cm and 30cm wide panels were cut from the 91cm wide panel after the fracture test of the 91cm wide panel was complete. The notches were made using a machining process called electronic discharge machining (EDM).

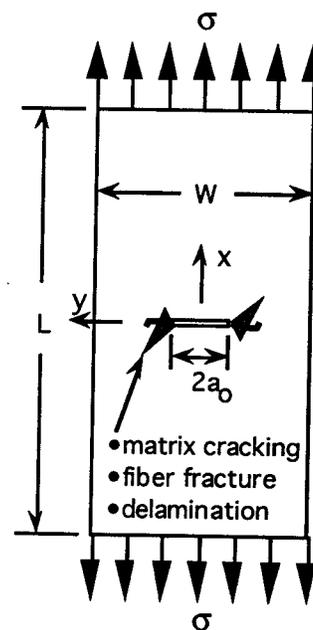


Figure 1. Center-Crack Tension Panel.

The 10cm wide notched panels were monotonically loaded to failure in a 223 kN servo-hydraulic testing machine. The 30cm and 91cm wide panels had anti-buckling guide plates attached just above and below the notch. They were loaded to failure in a 445 kN and 2225 kN servo-hydraulic testing machine, respectively. Specimens were strain gaged and the strain data and applied load from the load cell were recorded. A ring gage was secured in the center of the notch and the center-crack opening displacement from the ring gage was also recorded.

As the panels were being loaded, the subcritical microcrack damage (fiber fracture, delamination, and matrix cracking local to the notch tip) was frequently audible. Periodically, as damage progressed with increasing load, a zinc-iodide dye penetrant was applied to the notch and edge of the specimen. X-ray radiographs were taken of the right and left notch-tip regions. The damage absorbed the zinc-iodide dye penetrant and the damage is accurately represented in the

x-ray radiograph as a blackened or shaded region, Figure 2. The local delamination, matrix cracking, and fiber fracture were made visible by the dye penetrant. No damage initiated and progressed from the edge of the panels.

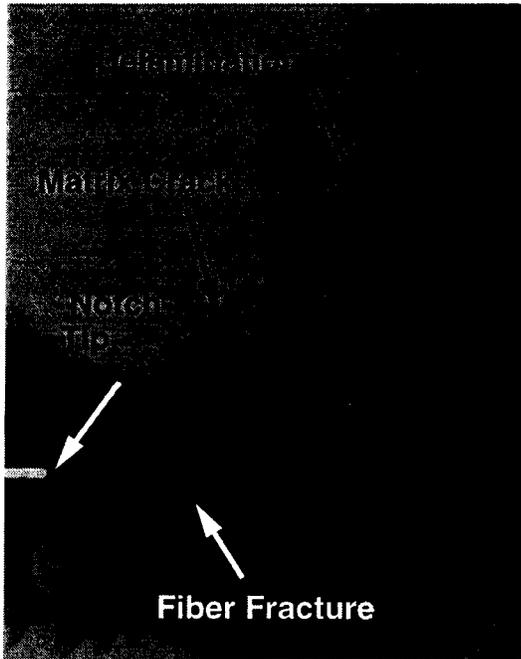


Figure 2. Notch-Tip Damage of an AS4/938 Center-Crack Tension Panel.

#### Quantifying Damage Growth

During the monotonic loading of these panels, the applied load and the crack opening displacement (COD) at the center of the notch was recorded and used to produce load/COD plots, as shown in Figure 3 for the AS4/938 specimen D1AK5A. Discontinuities, or *jumps*, exist at various places along the load/COD plot where the fiber fracture was audible during loading. At these discontinuities, the specimen was unloaded to take an x-ray and are labeled A, B, C, and D on the load/COD plot. It was observed that for each consecutive loading, additional damage did not occur until the loads at which previous damage had occurred were exceeded. The corresponding x-ray radiographs are given in Figure 4 to illustrate the amount of damage at each discontinuity. The x-ray radiographs are actual size. The effective damage growth can be measured directly from these x-ray radiographs, or the load/COD plot can be used to provide a closed form solution for determining the characteristic half-crack length,  $a$ . From  $a$ , the effective damage growth is determined as

$$\Delta a = a - a_0 \quad (1)$$

The closed form elasticity solution [8] for an isotropic material under plane stress is

$$a = \frac{\text{COD}}{4\epsilon_x} \quad (2)$$

where  $\epsilon_x$  is the uniaxial far-field strain, and the initial notch length is small compared to the panel width such that finite width effects are neglected. It can be shown that the closed form elasticity solution for determining the characteristic half-crack length for an orthotropic material under plane stress is

$$a = \frac{\text{COD} \cdot \zeta}{4\epsilon_x} \quad (3)$$

where  $\zeta$  represents the effects of finite width and anisotropy.

The effective damage growth,  $\Delta a$ , and the critical stress responsible for the damage growth was used to plot  $K_I$  vs.  $\Delta a$ . This plot is a damage growth resistance curve (R-curve), Figure 5. This figure displays damage growth resistance of three AS4/938 panels of the same geometrical configuration. The curve drawn through the data is the R-curve that was generated from the average of the three AS4/938 panels.

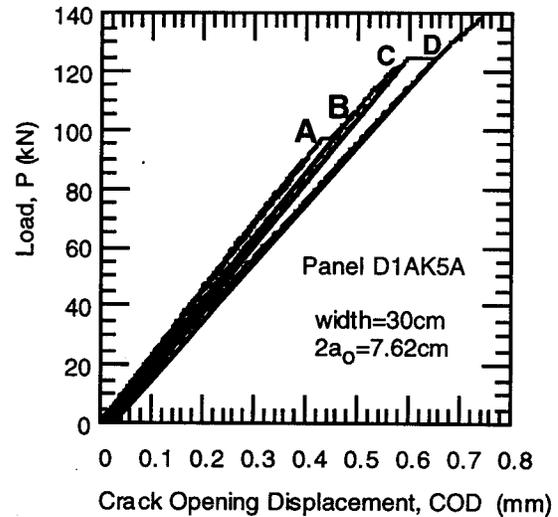


Figure 3. Load/COD Plot for Panel D1AK5A.

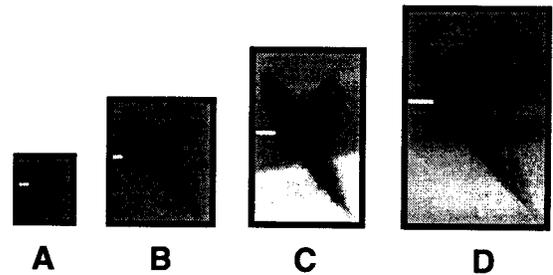


Figure 4. X-Ray Radiographs of Panel D1AK5A Notch-Tip Progressive Damage.

#### Progressive Damage Analysis

Observations of damage growth resistance in the experimental study led to an analytical study of R-curves using a progressive damage methodology. This methodology consists of a multi-purpose finite element code [9] and was recently developed to include a residual strength prediction capability [3]. The progressive damage model is damage dependent and can therefore model the damage development at and around the notch-tip. The progressive damage methodology is applicable to any laminate stacking sequence and the finite element analysis uses quadrilateral plate or shell elements to analyze general structural geometries.

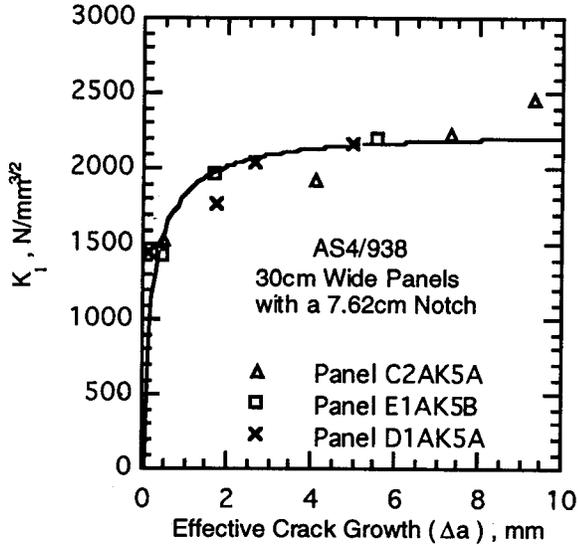


Figure 5. Experimental Damage Growth Resistance.

### Nonlinear Damage-Dependent Constitutive Model

The nonlinear, damage-dependent constitutive model of Allen and Harris [4-7] predicts the formation of intraply matrix cracks and fiber fracture for monotonic tensile loading and for tension-tension fatigue [10] (not discussed herein), the associated ply level stress and strain states, and the residual strength of laminates. In order to predict the initiation and growth of delamination, three-dimensional stress states such as those at free edges must be calculated. The analysis currently does not have this capability, therefore, delamination is not modeled in the present analysis.

The constitutive model uses internal state variables (ISV) to represent the average effects of local deformation due to the various modes of microcrack damage. This concept is called continuum damage mechanics. The constitutive model may be written as

$$\sigma_{ijL} = Q_{ijkl} \{ \epsilon_{kl} - \alpha_{kl} \}_L \quad (4)$$

where  $\sigma_{ijL}$  are the locally averaged components of stress,  $Q_{ijkl}$  are the ply-level reduced moduli, and  $\epsilon_{klL}$  are the locally averaged components of strain. The internal state variables,  $\alpha_{klL}$ , represent the local deformation effects of the various modes of damage. When the material is subjected to quasi-static (monotonic) loads, the incremental change of the internal state variable is assumed to be

$$d\alpha_{klL} = \begin{cases} f(\epsilon_{klL}, \beta, \gamma, \psi) & \text{if } \epsilon_{klL} > \epsilon_{klcrit}; \\ 0 & \text{if } \epsilon_{klL} < \epsilon_{klcrit} \end{cases} \quad (5)$$

where  $\epsilon_{klcrit}$  is the critical tensile failure strain and  $\beta, \gamma$ , and  $\psi$  are scale factors that describe the load carrying capability of the material after the occurrence of mode I (opening mode) matrix cracking, fiber fracture, and mode II (shear mode) matrix cracking, respectively. The physical interpretation of equation (5) is as follows: As long as the strains in a material element (local volume element or finite element) are less than the critical strains,  $\epsilon_{klcrit}$ , no damage exists and the internal state variables have a zero value. When the strains reach their critical value, the element is damaged and this

damage is represented by an internal state variable whose value is proportional to the local strain. The proportionality is dependent on the scale factors  $\beta$ ,  $\gamma$ , and  $\psi$ . Based on these assumptions, when fiber fracture, mode II matrix cracking, or mode I matrix cracking occur in a ply within an element, the longitudinal, shear, and transverse stresses for that ply in that element are

$$\sigma_{11} = \gamma S_{cr}^x \quad (6)$$

$$\sigma_{12} = \psi S_{cr}^{xy} \quad (7)$$

$$\sigma_{22} = \beta S_{cr}^y \quad (8)$$

where  $S_{cr}^x$ ,  $S_{cr}^{xy}$ , and  $S_{cr}^y$  are the lamina longitudinal, shear, and transverse critical strengths, respectively.

### Experimental and Computational Results

Damage growth resistance for the 91 cm panel is plotted in Figure 6 where the experimental data is compared to the model prediction. The experimental R-curve is the average of the experimental data shown in the plot. The predicted data points are represented by the dark filled circles. The figure shows that the model predicts stable damage growth, and the trend of the analytical results is similar to the experimental R-curve.

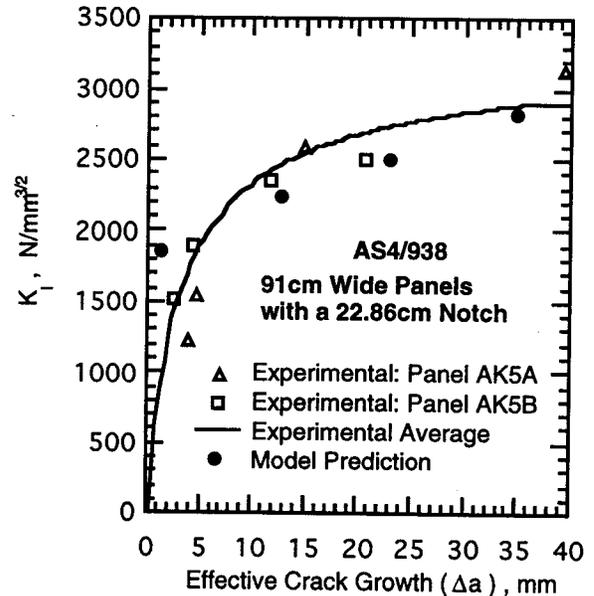


Figure 6. Experimental and Model Generated R-Curves.

### Concluding Remarks

Damage growth resistance of center-crack tension composite panels has been discussed and R-curves were used to illustrate this resistance. The composite panels are fiber-reinforced laminated composites with a through-penetration slit machined at the center and perpendicular to the applied tensile loads. Much like metallic materials, these composite panels exhibited resistance to damage growth which could be illustrated in the form of R-curves. Unlike plasticity in metals, the toughening mechanisms responsible for damage growth resistance in composites are due to micro-cracking. The matrix cracking, fiber fracture, and delaminations interact to redistribute the load and cause a

toughening effect which is manifested in damage growth resistance curves.

An experimental investigation was conducted where the center-crack panels were loaded in monotonic tension. X-ray radiographs were produced to illustrate the periodic extensions of damage. Load/COD plots were generated from the data and offer as much evidence of damage growth resistance as do the x-ray radiographs. This evidence was then put into a graphical representation known as an R-curve. The effective damage growth,  $\Delta a$ , could be determined by an elasticity solution involving COD measurements.

A progressive damage methodology was used to model the damage at and around the notch-tip. This analysis accounted for mode I and II matrix cracking and fiber fracture only. Damage growth resistance was predicted and the trends predicted by the analysis compared very well to the experimental R-curves.

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