Subject: FORUM ACUSTICUM 1999 Abstract

Acoustic Energy Estimates in Inhomogeneous Moving Media

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In ducted fan engine noise research, there is a need for defining a simple and easy to use acoustic energy conservation law to help in quantification of noise control techniques. There is a well known conservation law relating acoustic energy and acoustic energy flux in the case of an isentropic irrotational flow. Several different approaches have been taken to generalize this conservation law. For example, Morfey [JSV, 197 1] finds an identity by separating out the irrotational part of the perturbed flow. Myers [JFM, 1986] is able to find a series of indentities by observing an algebraic relationship between the basic conservation of energy equation for a background flow and the underlying equations of motion. In an approximate sense, this algebraic relationship is preserved under perturbation. A third approach which seems to have not been pursued in the literature is a result known as Noether's theorem. There is a Lagrangian formulation for the Euler equation of fluid mechanics [M. E. Taylor, PDE'S, 1996]. Noether's theorem says that any group action that leaves the Lagrangian action invariant leads to a conserved quantity. This presentation will include a survey of current results regarding acoustic energy and preliminary results on the symmetries of the Lagrangian. (M. E. Taylor, Partial Differential Equations III: Non-linear Equations, Springer-Verlag, 1996)

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Outline of the Talk

- – Origin of the problem
- – What is a conservation law?
- – A simple proof of nonuniqueness of energy conservation laws
- – Previous work on this subject
- – Present approaches to derivation of conservation laws
- – Some important results
- – Godin's method
- – Möhring's method
- – Noether's theorem
- – Concluding remarks

Origin of the Problem

Our **advanced method of ducted fan noise prediction** uses a system of coupled computer codes as follows:

– High resolution **unsteady aerodynamics** for rotor-stator interaction (CFD)

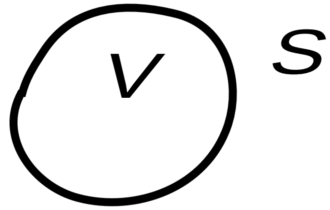
– **Wave propagation** in the duct (FEM/CFD)

– **Noise radiation** from duct inlet and exhaust

Our **objective** in the present study was to use a conservation law to check the accuracy of CFD calculations for wave propagation

What Is a Conservation Law?

An **energy conservation law** is defined by a relation of the forms:



$$\frac{d}{dt}\int_V w\,dV + \int_S \boldsymbol{I}\cdot dS = 0 \quad \text{Integral Form}$$

$$\frac{\partial w}{\partial t} + \nabla\cdot\boldsymbol{I} = 0 \quad \text{Differential Form}$$

Conservation laws are not unique.

A Simple Proof of Nonuniqueness of Energy Conservation Laws

Let $\boldsymbol{f}(\boldsymbol{x}, t)$ be a differentiable vector function. Let w and $\boldsymbol{I}$ satisfy an energy conservation law. Then

$$\frac{\partial(w + \nabla \cdot \boldsymbol{f})}{\partial t} + \nabla \cdot \left(\boldsymbol{I} - \frac{\partial \boldsymbol{f}}{\partial t}\right) = \frac{\partial w}{\partial t} + \nabla \cdot \boldsymbol{I} = 0.$$

Therefore $w + \nabla \cdot \boldsymbol{f}$ and $\boldsymbol{I} - \partial \boldsymbol{f} / \partial t$ also satisfy a conservation law. We can also add a linear combination of mass continuity or momentum equations of the right order to a conservation law to get a new conservation law.

Previous Work on This Subject

Many work has been published previously by **Blokhintsev, Chernov, Cantrell and Hart, Hays, Morfey, Möhring, Myers, Godin** and others.

Good sources for this subject are in the following books:

– Lighthill, Waves in Fluids

– Pierce, Acoustics

– Goldstein, Aeroacoustics

Present Approaches to Derivation of Conservation Laws

Currently there are two approaches:

1. Perturbation expansion of mass continuity, momentum and energy equations about the mean flow and searching for conservation laws mainly by intuitive reasoning (Blokhintsev, Morfey, Myers, Godin, etc.)

2. variational or Hamilton's principle and Clebsch potentials (Möhring), a method of great generality resulting in conservation laws that can not always be physically explained

Some Important Results

Assuming steady background flow and writing each fluid parameter as

$$q(\boldsymbol{x}, t) = q_0(\boldsymbol{x}) + \varepsilon q_1(\boldsymbol{x}, t) + \varepsilon^2 q_2(\boldsymbol{x}, t) + \ldots,$$

we can show that the conservation of energy laws for homentropic flows are:

$$\nabla \cdot [\rho_0 \boldsymbol{u}_0 (h_0 + u_0^2/2)] = 0$$

There is no first order energy conservation law.

Some Important Results (Cont'd)

In **Blokhintsev's conservation law**

$$\boldsymbol{I} = (c_0^2\rho_1/\rho_0 + \boldsymbol{u}_0 \cdot \boldsymbol{u}_1)(\rho_0\boldsymbol{u}_1 + \rho_1\boldsymbol{u}_0)$$

$$w = c_0^2\rho_1^2/2\rho_0 + \rho_1 u_0^2/2 + \rho_1\boldsymbol{u}_0 \cdot \boldsymbol{u}_1$$

If, however, there is transfer of energy between the mean and first order flow by vorticity ξ, then **Myers' result** is

$$\frac{\partial w}{\partial t} + \nabla \cdot \boldsymbol{I} = \rho_0\boldsymbol{u}_0 \cdot (\xi_1 \times \boldsymbol{u}_1) + \rho_1\boldsymbol{u}_1 \cdot (\xi_0 \times \boldsymbol{u}_0)$$

Some Important Results (Cont'd)

The **exact energy corollary of Myers** is:

$$\frac{\partial w^{\mu}}{\partial t} + \nabla \cdot \boldsymbol{I}^{\mu} = -(\rho \boldsymbol{u} - \rho_0 \boldsymbol{u}_0) \cdot (\xi \times \boldsymbol{u} - \xi_0 \times \boldsymbol{u}_0)$$

where H is the stagnation enthalpy and

$$w^{\mu} = \rho(H - H_0) - (p - p_0) - \rho_0 \boldsymbol{u}_0 \cdot (\boldsymbol{u} - \boldsymbol{u}_0)$$

$$\boldsymbol{I}^{\mu} = (\rho \boldsymbol{u} - \rho_0 \boldsymbol{u}_0)(H - H_0)$$

Godin’s Method

– **The Basic Idea:** Staring with the nonlinear governing equations of hydrodynamics and BC’s, a particular choice of **mixed Eulerian-Lagrangian** representation of fluid motion is used. The **oscillatory displacement** as a dependent variable simplifies drastically the linearized equations of hydrodynamics and the BC’s.

– **Godin** derives the **acoustic energy conservation law** by the inspection of these equations under rather general assumptions about the medium and the flow inhomogeneity.

– Godin has applied this method to fluids with **general equation of state and internal interfaces** as in ocean acoustics, **acoustic-gravity waves in fluids** and **mechanical waves in fluid-structure interaction**.

Möhring's Method

– **The Basic Idea:** A variational formulation can lead to a conservation law if the Lagrangian has a particular variable dependence, e.g., x, $\dot{x}$ or t missing.

– In general, there is no unique variational formulation. A variational formulation may have extraneous solution.

– **Seliger and Whitham** (1968) searched for the simplest variational formulation for mechanics of continua with unique solution using Clebsch potentials.

– Starting with the Lagrangian of Seliger and Whitham, **Möhring** found the Lagrangian for the perturbation of steady flow. The new Lagrangian did not have time t explicitly. This led to a **conservation law** which is not unique. These laws can be related using Hamiltonian formalism.

Noether's Theorem

If a dynamical system is gauge invariant under the infinitesimal transformation

$$\begin{cases} \bar{x}^i = x^i + \varepsilon F^i(t, \mathbf{x}, \dot{\mathbf{x}}) \\ \bar{t} = t + \varepsilon f(t, \mathbf{x}, \dot{\mathbf{x}}) \end{cases} \quad ; \; F^i - \dot{x}^i f \neq 0$$

with a Lagrangian L and a gauge function $P(t, \mathbf{x}, \dot{\mathbf{x}})$, then we have the **conservation law**:

$$P - \frac{\partial L}{\partial \dot{x}^i} F^i - \left(L - \frac{\partial L}{\partial \dot{x}^i} \dot{x}^i \right) f = 0$$

Noether's Theorem (Cont'd)

– There may be **new conservation laws** derived from Möhring's Lagrangian if we can find infinitesimal transformations under which the Lagrangian is invariant.

– We do not know whether these new laws are included among those obtained by Möhring using Hamiltonian formalism.

– There is extensive work on the **inverse variational problem** and conservation laws of dynamical systems which leads us to believe that new laws may exist.

Concluding Remarks

- There are **many energy conservation laws** for wave propagation in a moving medium available with various assumptions.
- The Hamiltonian formalism of **Möhring** and the mixed Eulerian-Lagrangian description of motion of **Godin** are at present the most general approaches in deriving conservation laws in acoustics.
- The application of **Noether's Theorem** in deriving conservation laws in acoustics of moving media appears to be a promising approach.
- For our ducted fan problem, **Myers exact energy corollary** is most suitable to check CFD results in the inlet duct but not in the region between the rotor and the stator because of the complex energy transfer between acoustic and vortical modes.