The Detection of Radiated Modes From Ducted Fan Engines

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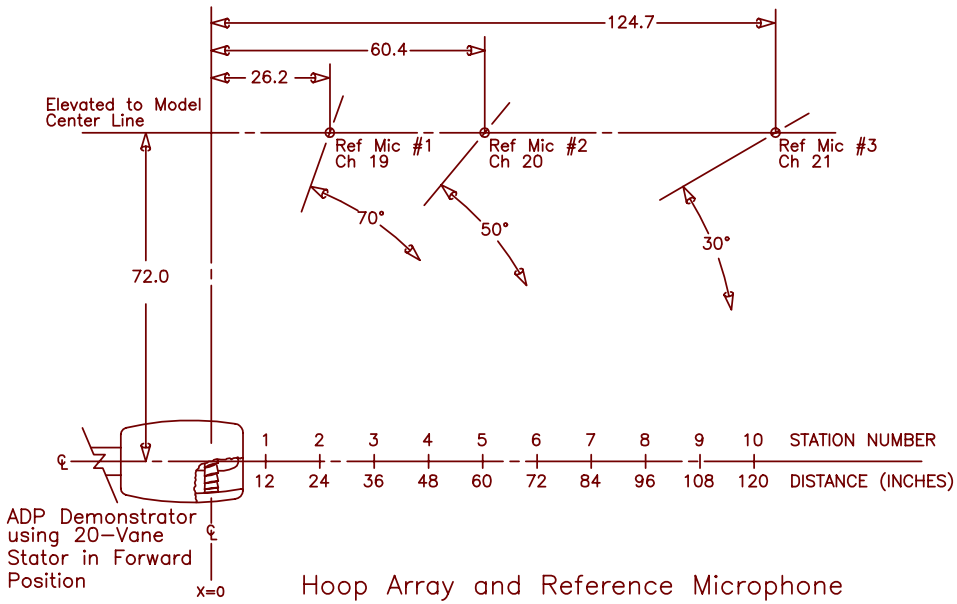
Outline of The Talk

- Introductory remarks
- Mode detection theory
- Ill-posedness and remedy
- Some results from a test
- Concluding remarks

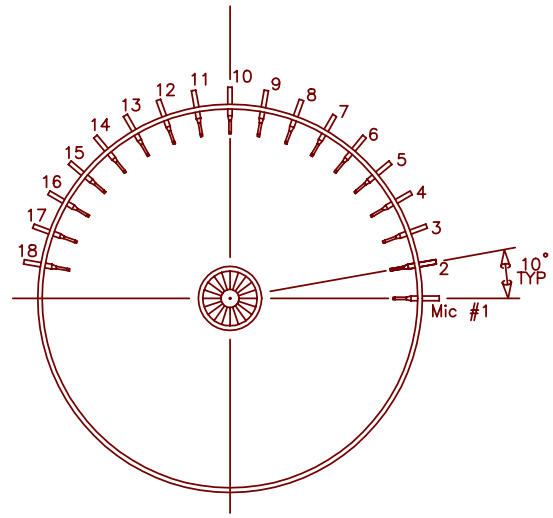
Introductory Remarks

- We present a nonintrusive method of detecting modes radiated from ducted fans by acoustic measurements in the far field
- A mode detection method is used to reduce engine noise or to help the designers to specify liners or active control
- Currently the only other mode detection method utilizes a rotating microphone rake which produces a wake at the inlet interacting with fan blades

Mode Detection Theory



Hoop Array and Reference Microphone Positioning Geometry
Top View



Hoop Array Microphones 1 through 18
ADP Demonstrator Model
Front View

NASA–LaRC Building 1218A Anechoic Noise Research Facility
Azimuthal Directivity Test of ADP Demonstrator
with NASA 9–Vane Inflow Control Device

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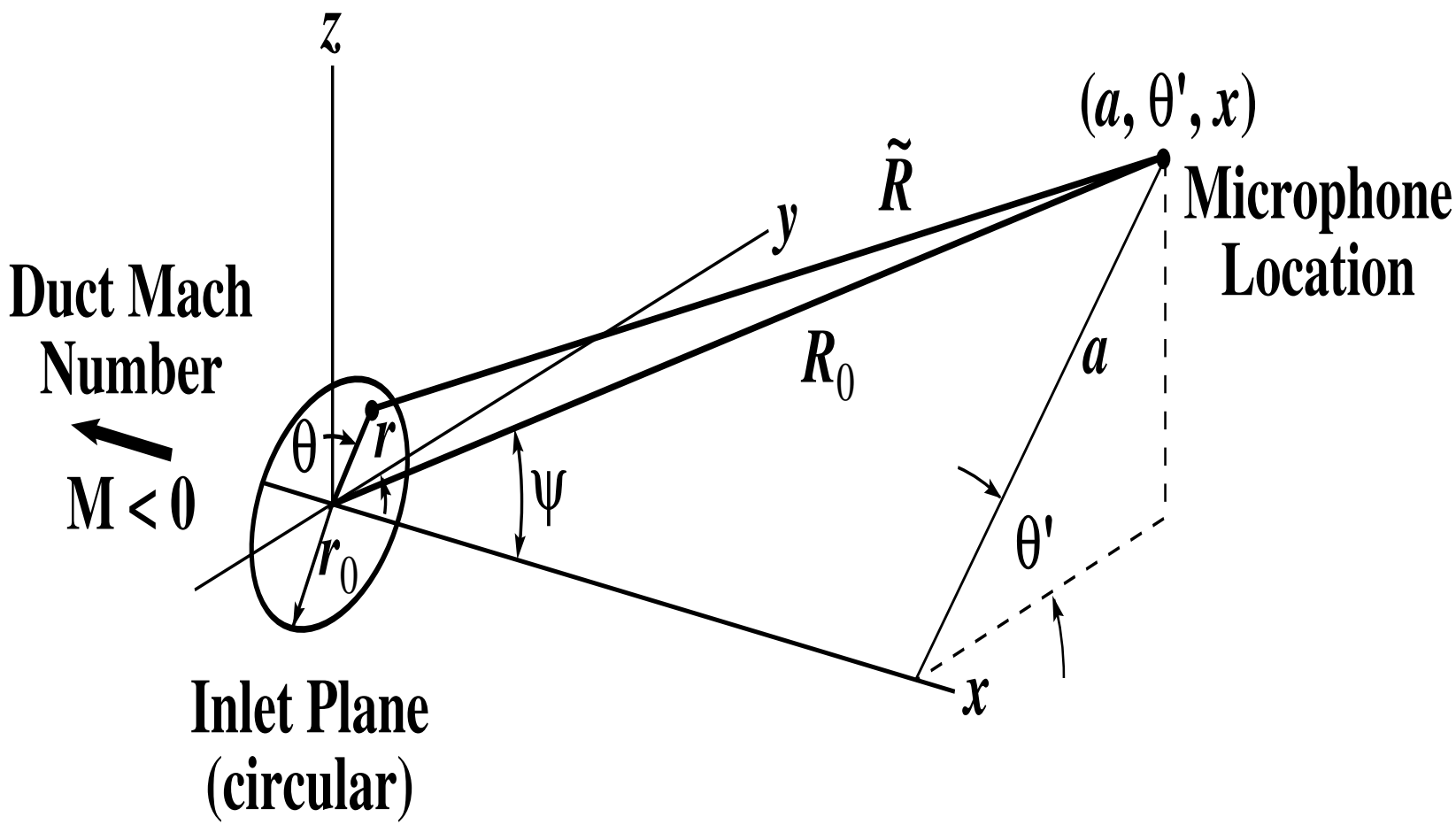
Mode Detection Theory



Mode Detection Theory

We will show that the acoustic pressure in the far field satisfies the following relation

$$p'(\boldsymbol{x}, t) \propto \frac{e^{-ikR_o}}{R_o} A(\theta', \psi) e^{i\alpha B(\Omega t - \theta')}$$

The information about mode amplitudes are hidden in the term $A(\theta', \psi)$. Note independence from rotor speed in this term.

$k = \alpha B \Omega / c$, B **fan blade no.,** α **multiple of BPF**

Mode Detection Theory

The acoustic pressure at the inlet plane for V vanes and B fan blades is

$$p'(r, \theta, x, t) = e^{i\alpha B(\Omega t - \theta)} \times \sum_{n} \sum_{q} A_{qn} J_m[k_r(m, n)r] e^{-iqV}$$

$m = \alpha B + qV$ circumferential mode no., n radial mode no., $q = 0, \pm 1, \pm 2, \ldots$ an index for circumferential mode no., k_a axial wave number, k radial wave number

Mode Detection Theory

Radiation model: piston in the wall

$$4\pi p'(\boldsymbol{x}, t) = \int_{\substack{inlet \\ area}} \frac{\rho_0 [\dot{u}]_{ret}}{\tilde{R}} dS$$

In the far field $\tilde{R} \approx R_0 - r \sin\psi \cos(\theta - \theta')$, and using $dS = r dr d\theta$, $\dot{u}(m, n) = i k_a(m, n) p'(m, n) / \rho_0$, we can integrate the above integral in θ and find the acoustic pressure in the far field as follows.

Mode Detection Theory

$$p'(\boldsymbol{x}, t) = \frac{i}{2R_0} e^{i\alpha B[\Omega(t - R_0/c) - \theta' + \pi/2]} \times$$
$$\sum_n \sum_q C(m, n, \psi) A_{qn} k_a(m, n) e^{-iqV(\theta' - \pi/2)}$$

$$C(m, n, \psi) = \int_0^{r_0} r J_m(kr \sin\psi) J_m[k_r(m, n) r] dr$$
$$= \frac{J_m(kr_0 \sin\psi) J_m[k_r(m, n) r_0]}{k_r^2(m, n) - k^2 \sin^2\psi}$$

Mode Detection Theory

Let $\hat{p}'(\boldsymbol{x}, \omega)$ be the temporal Fourier transform of $p'(\boldsymbol{x}, t)$, define

$$P(\boldsymbol{x}, \alpha B\Omega) = -2iR_0 \hat{p}'(\boldsymbol{x}, \alpha B\Omega) \times e^{i\alpha B(\Omega R_0/c + \theta' - \pi/2)}$$

The spatial Fourier transform of this in azimuthal direction is

$$\hat{P}_q = \frac{1}{\pi}\int_0^{\pi} P(\boldsymbol{x}, \alpha B\Omega) e^{iqV\theta'} d\theta'$$

$$= e^{i\pi qV/2} \sum_n C(m, n, \psi) k_a(m, n) A_{qn} = e^{i\pi qV/2} D_q$$

Mode Detection Theory

This means that for any fixed axial position of the array, i.e., for a fixed ψ, we have one linear relation between the amplitudes of all radial modes associated with a given circum. mode $m = \alpha B + qV$.
This linear relation is

$$D_q = \sum_n C(m, n, \psi) k_a(m, n) A_{qn}$$

Changing the angle ψ gives us more equations to form a system of linear equations for the mode amplitudes.

Ill-Posedness and Remedy

We have an overdetermined system of linear equations

$$\boldsymbol{G} A_q = d_q$$

where $\boldsymbol{G}$ is a $j \times n$ matrix such that $j \geq n$,

$A_q = (A_{q1}, A_{q2}, \ldots, A_{qn})^T$ and

$d_q = (D_q(\psi_1), D_q(\psi_2), \ldots, D_q(\psi_j))^T$

This system of equations is not compatible because of measurement and numerical errors. The problem of mode detection is ill-posed because the matrix $\boldsymbol{G}$ has very small singular values.

Ill-Posedness and Remedy

To find meaningful solution for the amplitudes, we must use a regularization technique such as singular value decomposition (SVD) or Tikhonov regularization. We have selected Tikhonov regularization. The solution minimizes the following cost function $\|\boldsymbol{G}A_q - d_q\|_Y + \alpha_T \|A_q\|_X$, where α_T is a parameter of the order of the square of the smallest nonzero singular values.

See:Rainer Kress: Numerical Analysis, Springer, 1998

Some Results From a Test

$B = 16$, $r_0 = 0.1524\ m\ (6\ in)$, $M = 0.605$,
$V = 20$, $RPM = 16,900\ (BPF = 4506\ hz)$,
$a = 0.912\ m\ (3\ ft)$, No. of microphone measurements over a half-circle $= 360$,
Number of axial stations measured $= 10$

Possible interaction modes:

BPF: (-4,1) to (-4,4)
2BPF: (-28,1), (-8,1) to (-8,7), (12,1) to (12,5)
3BPF: (-32,1) to (-32,3), (-12,1) to (-12,10), (8,1) to (8,12), (28,1) to (28,4)

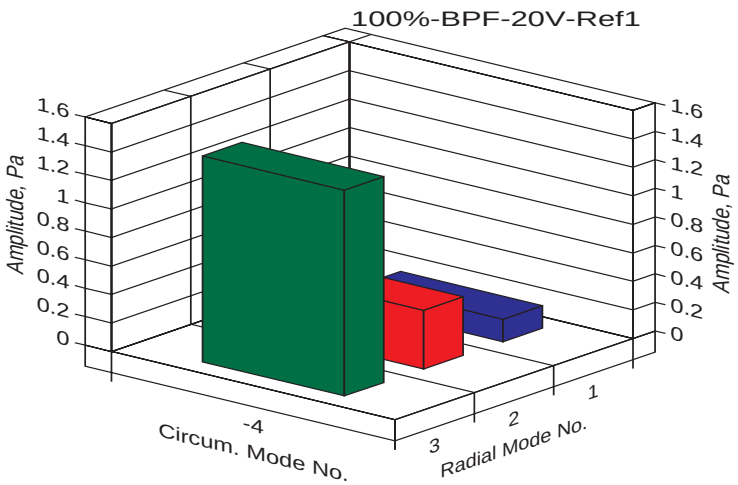
Some Results From a Test

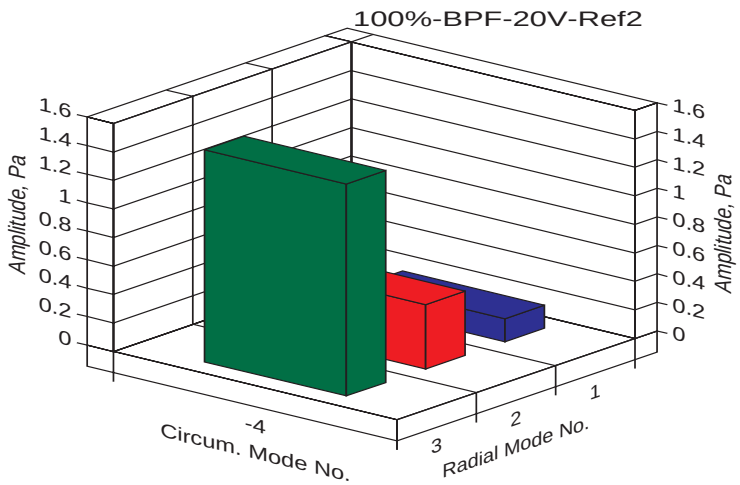
An indication of ill-posedness of mode detection problem: Singular values of the matrix G for circumferential mode -12 at 3BPF

μ_i : *4.42E-4, 3.63E-4, 2.98E-4, 2.37E-4*
2.10E-4, 1.73E-4, 5.58E-5, 9.89E-7
3.44E-9, 1.52E-12

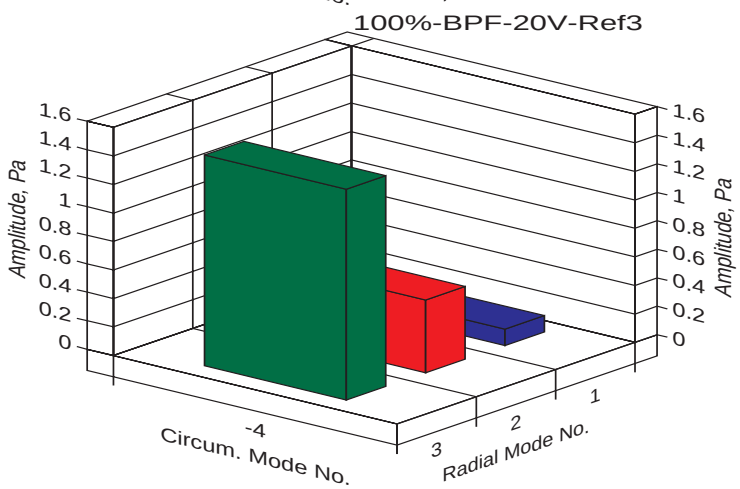
The matrix G^*G used to solve for mode amplitude is highly ill-conditioned. We use Tikhonov regularization with $\alpha_T = 10\text{E-}10$.

Some Results From a Test





BPF Results



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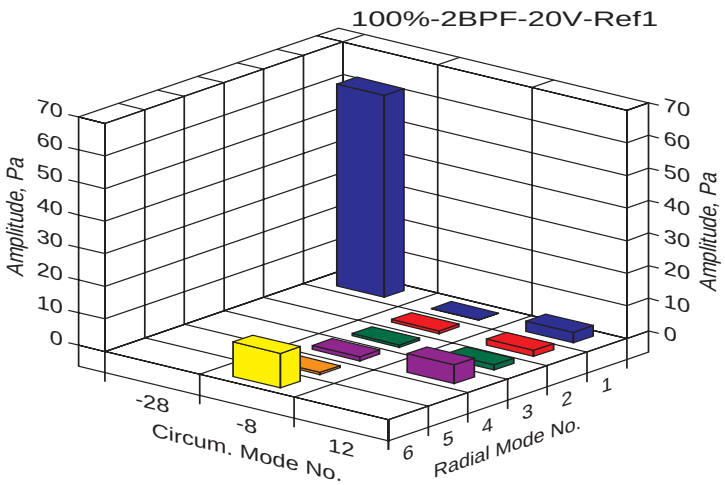
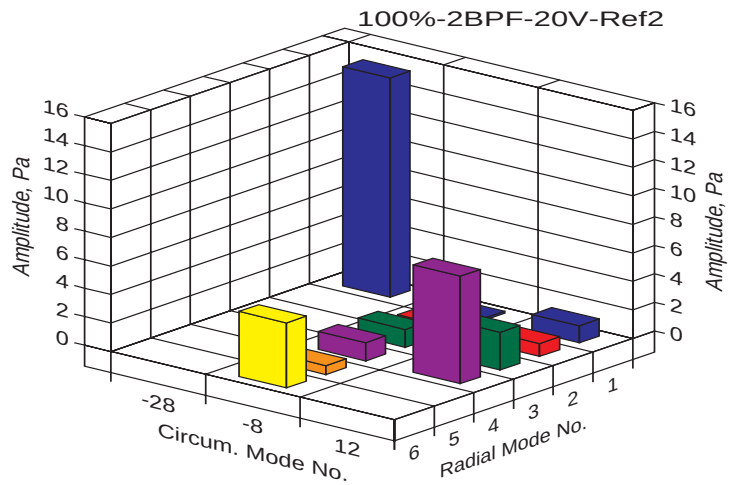
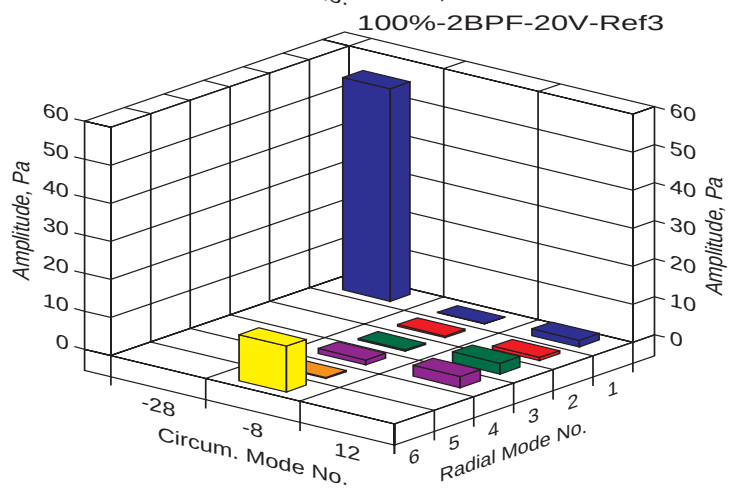
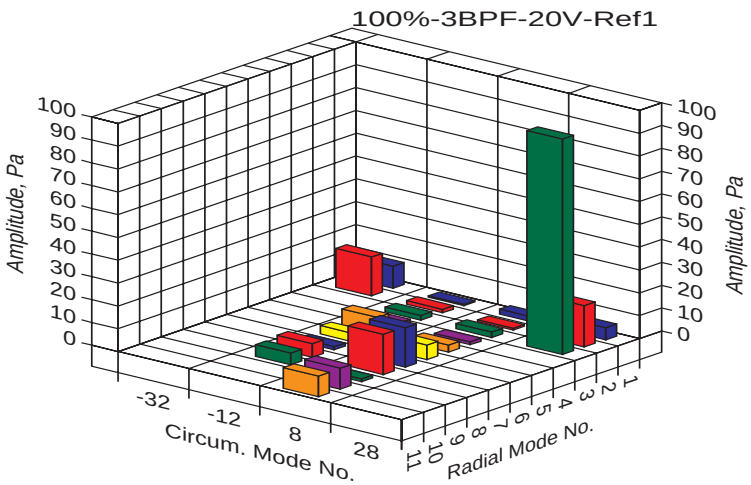


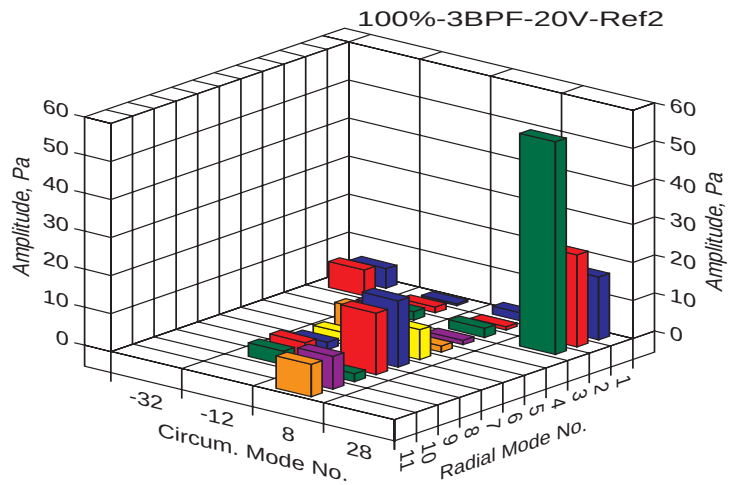


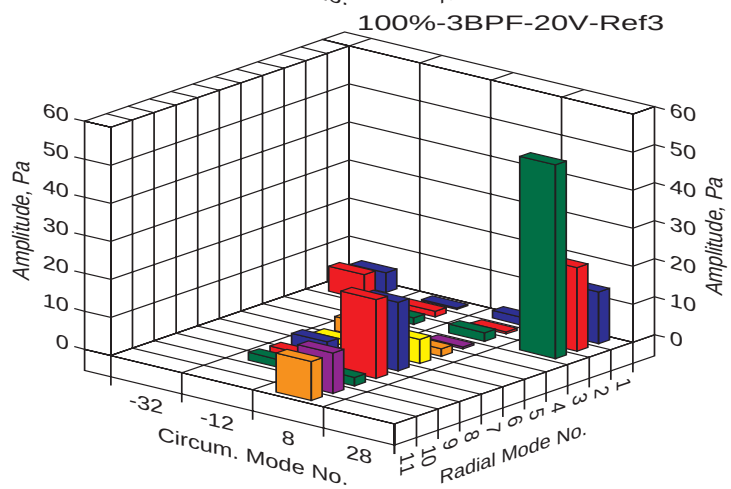
2 BPF Results



Some Results From a Test







3 BPF Results

Concluding Remarks

- Our mode detection technique is highly suitable for finding the amplitudes of radiated modes from a ducted fan
- Amplitudes of modes near cut off can be found by more measurements near inlet
- Many improvements are possible