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Abstract

The thermal model for transverse heat flow of having single filament in a unit cell is extended. In this model, we proposed that two circular filaments in a unit cell of square packing array and obtained the transverse thermal conductivity of an unidirectional material.

I. Thermal Model

Predicting equivalent thermal conductivity of transverse flow in unidirectional composite materials is of interest in composite research. An expression for thermal conductivity can be obtained based on physical principle, either from the point of view of equivalent thermal flux or from equivalent temperature field. By considering the fibers as square slabs or circular cylinders embedded in the matrix, the simple rule of mixture can be applied to obtain the equivalent thermal conductivity. In 1967, Springer and Tsai [1] proposed a thermal model for transverse flow that is a compound model including both serial flow and parallel flow to account for the fiber shape geometry. The thermal model developed has a single circular filament located in the center of a square packing array. The resulting expression is in good agreement with experimental work. Recently, this was extended to an elliptical shaped fiber with the same cross section area as the circular fiber. It was shown that the transverse composite thermal conductivity depends strongly on the fiber shape [2]. In this report, an expression is presented for a configuration with two circular fiber filaments in a unit cell of square packing array. The arrangement is shown as in Fig.1. One filament is located in the center of the cell and one quarter of the filament located at each of the four corners of the cell. Assuming that the radius of circular fiber is r, and the size of the unit cell is $4a^2$, since there are two filaments per cell, the volume fraction is

$$v_f = 2\pi r^2 / 4a^2 \qquad (1)$$

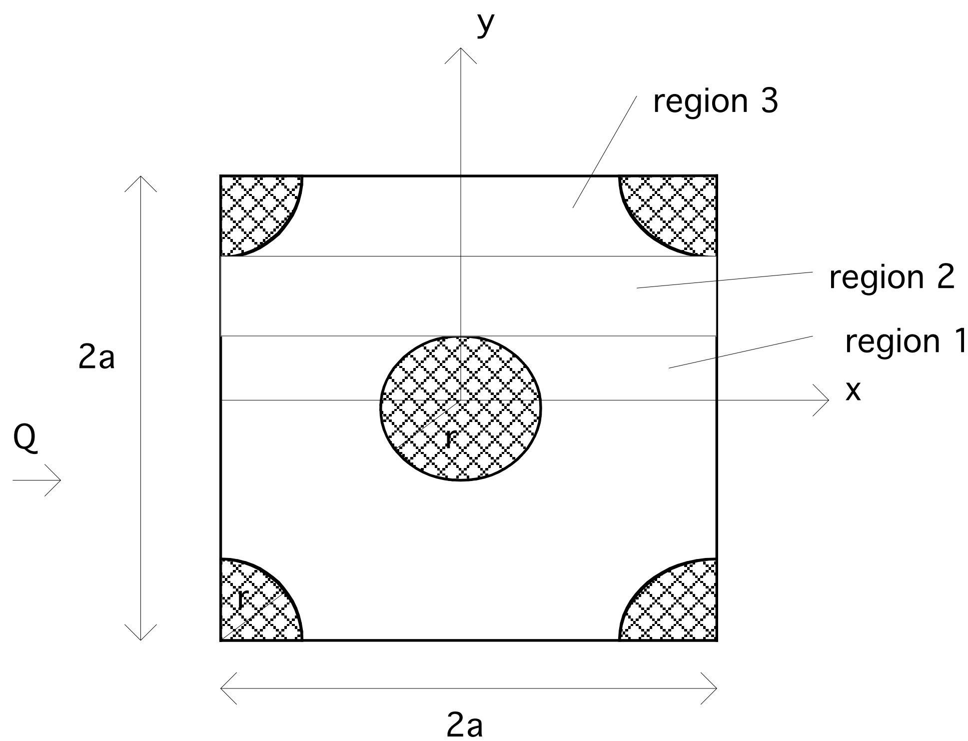

Fig . 1 Fundamental elements used in thermal modeling for fiber radius $r \leq a/2$ arrangement

The heat flow is from the left to right and Fig. 1 is symmetrical with respect to reflection in both x and y directions. To obtain an expression for the thermal conductivity, it is only necessary to consider the heat flow in the upper half of the cell. The upper cell is further divided into three regions with limits in the y direction, from 0 to r, r to $a-r$ and $a-r$ to a respectively, while the limit in x is from $-a$ to a for all regions. In region 2, there is only matrix; in regions 1 and 3, there is fiber as well as matrix. As far as the thermal model is concerned, the same amount of heat is conducted in region 1 and region 3, because regions 1 and 3 occupy the same area, and contain same shape of fiber. As the radius of fiber increases from 0 to $a/2$, region 2 decreases to 0, i.e., there is no direct passage for heat to traverse through the cell without intersecting the fiber. As r increases from $a/2$ to the maximum, $a/\sqrt{2}$ shown in Fig. 2, it is necessary to redefine the regions of interest. Region 1 is now redefined as $y=0$ to $y=a-r$; region 2 is redefined as $y=a-r$ to $y=r$; region 3 is redefined as $y=r$ to $y=a$. Following [1], the ratio of transverse thermal conductivity k_{22} to the matrix thermal conductivity k_m is written as

$$k_{22}/k_m = (1-\frac{2r}{a}) + 2\int_0^r \frac{dy}{a + x(1-\frac{k_m}{k_f})}$$

$$= (1-\frac{2r}{a}) + \frac{1}{B}[\pi - \frac{4}{\sqrt{1-\frac{r^2B^2}{a^2}}}\tan^{-1}\frac{1-\frac{rB}{a}}{\sqrt{1-\frac{r^2B^2}{a^2}}}] \qquad (2)$$

where k_f is the thermal conductivity of the fiber, $r \le a/2$, and $r^2B^2/a^2 \le 1$

$B = (1-\frac{k_m}{k_f})$ and $x^2 + y^2 = r^2$

However, the expression takes a different form when $r^2B^2/a^2 \ge 1$

$$k_{22}/k_m = (1-\frac{2r}{a}) + \frac{1}{B}[\pi - \frac{2}{\sqrt{\frac{r^2B^2}{a^2}-1}}\log\left|\frac{B\frac{r}{a}-1+\sqrt{\frac{r^2B^2}{a^2}-1}}{B\frac{r}{a}-1-\sqrt{\frac{r^2B^2}{a^2}-1}}\right|] \qquad (3)$$

Notice that B is defined differently than in Ref. [1]. As r increases beyond $a/2$, the ratio can be expressed as

$$k_{22}/k_m = 2\int_0^{a-r}\frac{dy}{B\sqrt{r^2-y^2}+a} + \int_{a-r}^{r}\frac{dy}{B(\sqrt{r^2-y^2}+\sqrt{r^2-(y-a)^2})+a} \qquad (4)$$

where $a/2 \le r \le a/\sqrt{2}$. Eq.(4) can be solved readily by numerical quadrature. Eqs.(2), (3) and (4) can be expressed in terms of volume fraction v_f by relation (1)

$$k_{22}/k_m = (1-\sqrt{\frac{8v_f}{\pi}}) + \frac{1}{B}[\pi - \frac{4}{\sqrt{1-\frac{2v_fB^2}{\pi}}}\tan^{-1}\frac{1-B\sqrt{\frac{2v_f}{\pi}}}{\sqrt{1-\frac{2v_fB^2}{\pi}}}] \qquad (5)$$

where $\sqrt{\frac{2v_f}{\pi}} \le \frac{1}{2}$, $\frac{2v_fB^2}{\pi} \le 1$

and

$$k_{22} / k_m = (1-\sqrt{\frac{8v_f}{\pi}}) + \frac{1}{B}[\pi - \frac{2}{\sqrt{\frac{2v_f B^2}{\pi}-1}} \log\left|\frac{B\sqrt{\frac{2v_f}{\pi}}-1+\sqrt{\frac{2v_f B^2}{\pi}-1}}{B\sqrt{\frac{2v_f}{\pi}}-1-\sqrt{\frac{2v_f B^2}{\pi}-1}}\right|]$$

$$\text{where } \sqrt{\frac{2v_f}{\pi}} \leq \frac{1}{2}\ , \ \ \frac{2v_f B^2}{\pi} \geq 1 \tag{6}$$

$$k_{22} / k_m = 2\int_0^{1-\sqrt{\frac{2v_f}{\pi}}} \frac{dz}{B\sqrt{\frac{2v_f}{\pi}-z^2}+1} + \int_{1-\sqrt{\frac{2v_f}{\pi}}}^{\sqrt{\frac{2v_f}{\pi}}} \frac{dz}{B[\sqrt{\frac{2v_f}{\pi}-z^2}+\sqrt{\frac{2v_f}{\pi}-(z-1)^2}]+1}$$

$$\text{where } \frac{1}{2} \leq \sqrt{\frac{2v_f}{\pi}} \leq \frac{1}{\sqrt{2}} \tag{7}$$

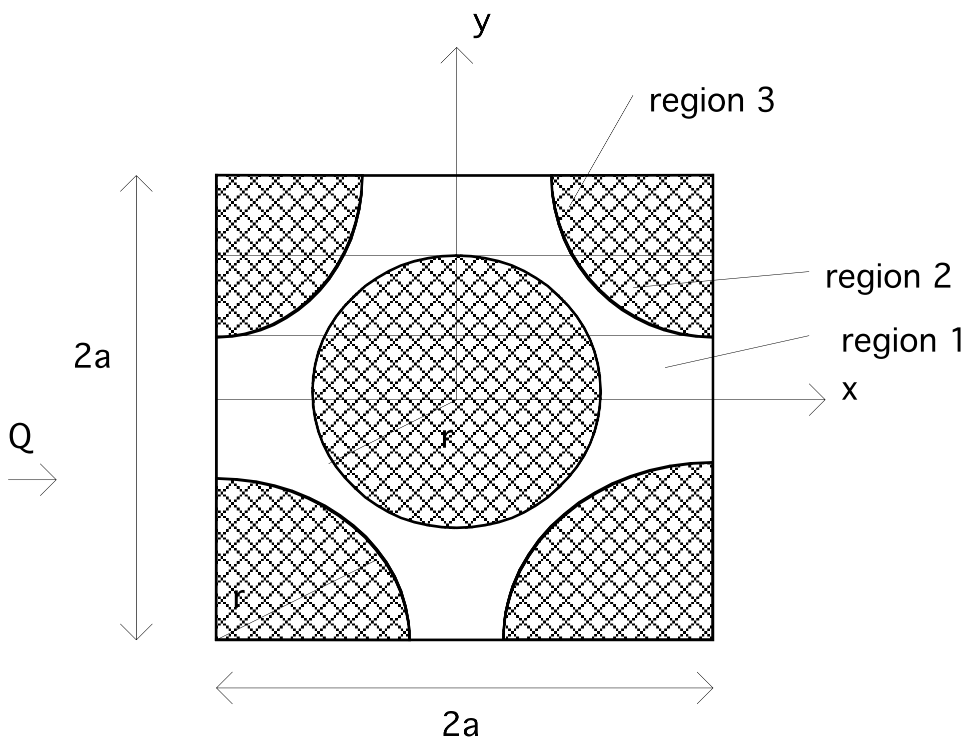

Fig . 2 Fundamental elements used in thermal modeling for fiber radius $r \geq a/2$ arrangement

II. Comparison of Results

In the following, Eqs. (5), (6) and (7) are used to compute the k_{22}/k_m ratio. These results are compared to results from reference [1] for the ratio determined as a function of fiber volume fraction v_f. Fig. 3 presents a comparison of the models and data for k_f/k_m equal to 666 and Fig. 4 presents results for k_f/k_m equal to 4.4 as was done in reference [1]. As expected, the 2 circular fibers per cell model predicts lower values for composite transverse conductivity than the single fiber models for the most of the range of v_f. The exception is for large k_f/k_m ratios and extremely high volume fraction where its value is higher than the value predicted by the single fiber square model, but lower than the single fiber circular cross section model. The lower values of two fiber model thermal conductivity can be explained in a qualitative sense; for the same v_f, the two fiber model has a more "serial" nature than the single fiber model, and therefore tends to predict a lower value. In an intuitive sense, introducing more fibers tends to further partition the matrix, so that the heat conduction suffers more impedance. On other hand, the two fiber model is a closer match to reality than the single fiber model because this arrangement allows the fibers to interlace together when the fiber radius increases beyond *a/2,* whereas the single fiber model definitely lacks of this characteristic. The experimental results of Thornburg and Pears [3] published in 1965, are shown by x's in Fig. 3 and 4. These results seem to be in better agreement with the single fiber model when the is k_f/k_m high, but in better agreement with the two fiber model when k_f/k_m is low. Of course, this is by no means conclusive, and we think this comparison really opens a new issue. What is clear is the predicted equivalent transverse thermal conductivity for a given value of volume fraction of the filaments depends upon the adopted models as clearly demonstrated here. To verify which is the correct model for predicting composite conductivity, a few carefully controlled experiments are required to establish the data base and enhance our understanding of the transverse heat conduction phenomenon in composite. From those experiments and theoretical models, we can gain an improved understanding of the mechanisms of heat conduction in composites and how to best expand the existing models for transverse composite conduction.

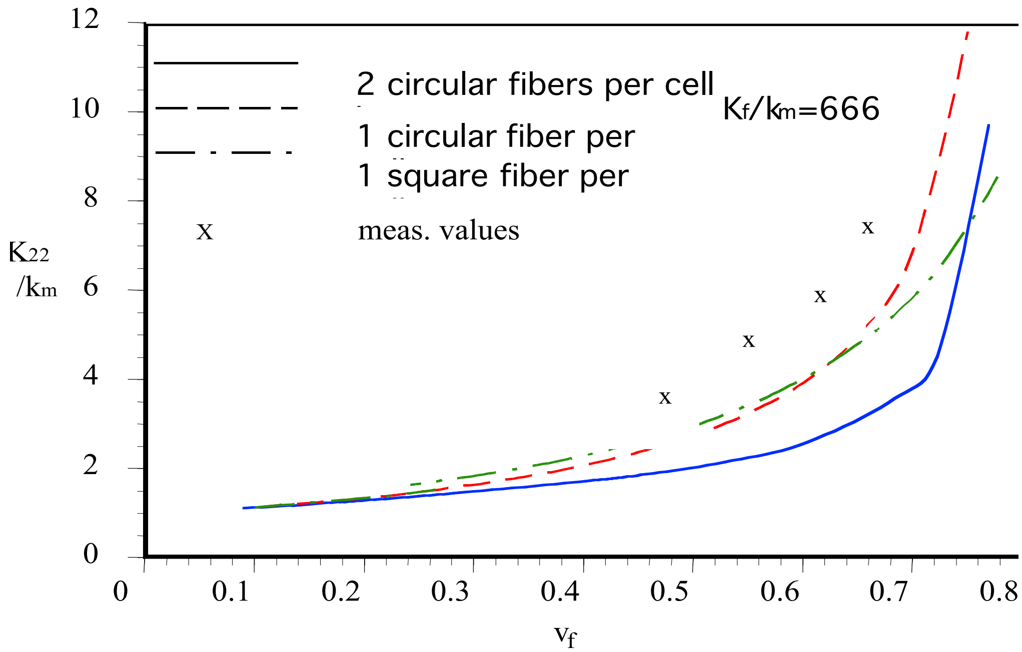

Fig. 3 Comparison of composite transverse thermal conductivity from different models for higher ratio of k_f/k_m.

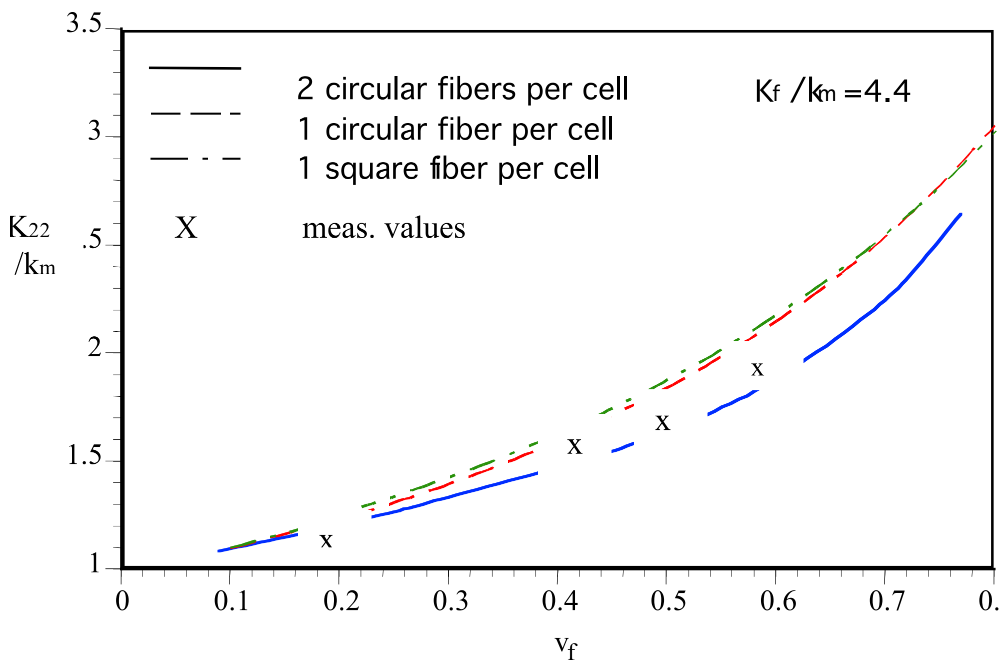

Fig. 4 Comparison of composite transverse thermal conductivity from different models for lower ratio of k_f/k_m.

Nomenclature

$2a, 2a$ = Fiber spacing in x and y direction
B = Dimensionless parameter , $B \equiv (\frac{k_m}{k_f} - 1)$
Q = average heat flux per unit cell length
v_f = volume fraction $=2\pi r^2/4a^2$
k_{22} = composite thermal conductivity the direction normal to the fibers
k_m = thermal conductivity of matrix
k_f = thermal conductivity of fiber in the direction normal to the fibers

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[1] G. S. Springer and S. W. Tsai, “Thermal conductivities of unidirectional materials”, J. Composite Materials, v.1, 166, 1967.
[2] H. Tai, “Dependence of the transverse thermal conductivity of unidirectional composites on fiber shape”, Int. J. Thermophysics, v.19, 5, p1485, 1998.
[3] J. D. Thornburg and C. D. Pears, “Prediction of the thermal conductivity of filled and reinforced plastics”, ASME Paper 65-WA/HT-4, 1965.

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