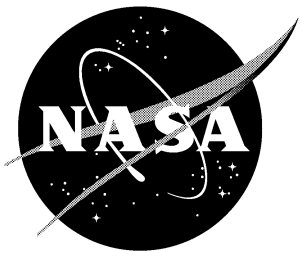

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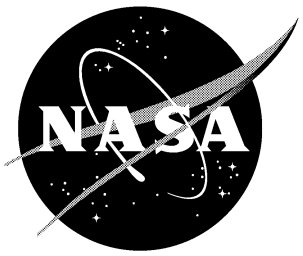

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Parameterized Spectral Distributions for Meson Production in Proton-Proton Collisions

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Abstract

Accurate semiempirical parameterizations of the energy-differential cross sections for charged pion and kaon production from proton-proton collisions are presented at energies relevant to cosmic rays. The parameterizations, which depend on both the outgoing meson parallel momentum and the incident proton kinetic energy, are able to be reduced to very simple analytical formulas suitable for cosmic-ray transport through spacecraft walls, interstellar space, the atmosphere, and meteorites.

Introduction

In order to solve cosmic-ray transport problems, a detailed understanding of the proton-proton interaction is needed. Galactic cosmic rays consist of approximately 85-percent protons (ref. 1) and solar cosmic rays consist almost entirely of protons and α-particles, whereas geomagnetically trapped particles in the Van Allen belts consist of protons and electrons (ref. 2). The interstellar medium itself consists primarily of hydrogen (protons) and helium (ref. 3). Hence, the proton-proton collision is the most common and most numerous interaction between cosmic rays and the interstellar medium, as well as within the solar system and the Earth's atmosphere.

Proton transport can be adequately modeled by using the Boltzmann equation in the straight-ahead approximation (ref. 2), which is given as

$$\left[\frac{\partial}{\partial x}-\frac{\partial}{\partial E}S(E)+\sigma\right]\phi(x,E) = \int_E^{\infty} f(E,E')\,\phi(x,E')\,dE' \quad (1)$$

where $S(E)$ is the proton stopping power, σ is the media macroscopic cross section, $\phi(x, E)$ is the particle flux density, $f(E, E')$ is the secondary-particle differential cross section in terms of secondary-particle energy (E) and proton energy (E'), and x is a scaling variable. This straight-ahead approach is used in current transport codes for particle propagation through various media (refs. 4–7).

One major shortcoming of the work done to date is that although the secondary-particle-production cross section $f(E, E')$ in equation (1) is taken to include all possible secondary particles, codes have not yet been run using meson cross sections (ref. 2). In this paper, parameterizations of cross sections for charged pions and kaons suitable for use in equation (1) will be provided.

The numerical approach currently used in solving equation (1) implements the cumulative energy spectrum, which is related to $f(E, E')$ above as follows (ref. 2):

$$F(r,r') = \int_0^{E'} f(E,E')\,dE \quad (2)$$

where the residual range (r) is given by $r = \int dE'/S(E')$. For a very thorough analysis of proton transport and a detailed explanation of the numerical procedure for solving equation (1), the reader is referred to Wilson et al. (ref. 2). Such an approach requires that the spectral distribution $f(E, E')$ be calculated a large number of times, thus making it impractical to utilize complicated analytical formulas for $f(E, E')$. Hence, the parameterizations derived herein will be presented in the simplest possible terms (without losing accuracy, of course).

An interaction cross section can be written in various forms, and high-energy physicists are interested in the Lorentz-invariant form $E\,d^3\sigma/d^3p$ (where σ is the total cross section and p is an energy momentum), which can be calculated from first principles using quantum field theory and is thus a convenient way to check theory with experiment. In this light, all early theoretical and experimental work done with mesons is presented in Lorentz-invariant form (refs. 8–11).

However, $f(E, E')$ from equation (1) is the energy-differential cross section $d\sigma/dE$ (or spectral distribution). The Lorentz-invariant cross section can still be used as it contains all the information one might need (including angular and spectral distributions as well as the total cross section). It now becomes a matter of retrieving this information.

In one of the early papers (ref. 11), all then-available accelerator data were gathered, and a very accurate parameterization of the Lorentz-invariant cross section was fit to the data. The aim of the current work is to utilize this representation of the Lorentz-invariant cross section in order to generate a representation of the spectral distribution that will then be parameterized as a simple function of incident proton energy and secondary meson momentum. The final result will provide the proper differential cross section for equation (1).

Symbols

A_1	parameter from table 1, mb/(GeV$^2/c^3$)
A_2, C, C_2	parameters from table 1, (GeV/c)$^{-1}$
C_1, γ	parameters from table 1, dimensionless

C_3	parameters from table 1, $(\text{GeV}/c)^{-2}$
c	speed of light
c.m.	center-of-mass frame
$d^3\sigma$	differential cross-section element, mb
d^3p	differential momentum element, GeV/c
E	energy, GeV
f	continuous spectrum $f(E, E')$ from equation (2)
K	kaon
M	$= m_A + m_B$ (in reaction, $A + B \rightarrow C + D$), GeV/c^2
m	mass, GeV/c^2
p_N	energy-momentum four vector
$\boldsymbol{p}$	momentum vector, GeV/c
q	parameter from equation (5)
r	residual range from equation (2)
$S(E)$	proton stopping power from equation (1)
s	Mandelstam energy variable
x	scaling variable used in representation of Lorentz-invariant cross section
$x_{//}^*$	ratio of c.m. momentum to maximum transferable momentum
α, β, χ	parameters in equation (14)
θ	scattering angle
σ	cross section, mb
ϕ	particle flux density from equation (1)

Subscripts:

max	maximum
p	proton
s	secondary particle
π	pion
$\perp$	perpendicular
//	parallel

Invariant Cross Section

The Lorentz-invariant cross section is given in terms of the following scaling variable (refs. 11 and 12):

$$\tilde{x} = \left[x_{//}^{*2} + \frac{4}{s}(p_\perp^2 + m_s^2) \right]^{1/2} \tag{3}$$

where $x_{//}^*$ is the ratio of the parallel component of the center-of-mass (c.m.) momentum to the maximum transferable momentum, $p_\perp$ and $p_{//}$ are the perpendicular and parallel components of the c.m. momentum, respectively, s is the Mandelstam energy of the system, and m_s is the secondary-particle rest mass.

In terms of this variable, the following parameterizations are then given (ref. 11) for pions:

$$\frac{E\,d^3\sigma}{d^3p} = \frac{A_1}{1 + (4m_p^2/s)^\gamma}(1-\tilde{x})^q \exp\left[\frac{-A_2 p_\perp}{1 + (4m_p^2/s)}\right] \tag{4}$$

where

$$q = \frac{C_1 + C_2 p_\perp + C_3 p_\perp^2}{1 + (4m_p^2/s)} \tag{5}$$

and for kaons:

$$\frac{E\,d^3\sigma}{d^3p} = A_1(1-\tilde{x})^C \exp(-A_2 p_\perp) \tag{6}$$

where the parameters A_1, A_2, C, C_1, C_2, C_3, and γ are given in table 1, and m_p is the proton rest mass.

Now, if the spectral distribution is to be written in simple terms (i.e., constants, mass, momenta, and energies of the interacting particles), then it will be desirable to first express the Lorentz-invariant cross section in such terms. Specifically, the Mandelstam energy variable (s) and the fractional momentum ($x_{//}^*$) must be simply represented.

The Mandelstam representation of the energy for any system $A + B \rightarrow C + D$ is defined as

$$s \equiv (\boldsymbol{p}_A + \boldsymbol{p}_B)^2/c^2 \tag{7}$$

where $\boldsymbol{p}_A$ and $\boldsymbol{p}_B$ are energy-momentum four vectors, given by $\boldsymbol{p}_N = E_N/c\boldsymbol{p}_N$, where the bold-faced $\boldsymbol{p}_N$ represents the usual three-dimensional momentum vector. This can then be rewritten as (see appendix A)

$$s = m_A^2 + m_B^2 + (2E_A m_B/c^2) \tag{8}$$

For the current problem, A and B are protons and E_A is the total incident proton energy (given by the sum of the proton kinetic energy (T_{lab}) and the rest mass (m_p). Thus, the following representation is given:

$$s = 4m_p^2 + 2m_p T_{\text{lab}} \tag{9}$$

The fractional momentum $x_{//}^*$ is given by $p_{//}/p_{\text{c.m.,max}}$ (ref. 11), where $p_{\text{c.m.,max}}$ is the maximum transferable momentum allowed by the kinematics in the center-of-mass frame. For the arbitrary system

Table 1. Parameters From Equations (4), (5), and (6) Used in Representation of Invariant Cross Section

[Data based on ref. 11]

Particle	A_1, mb/(GeV$_2$/c_3)	A_2, $(\mathrm{GeV}/c)^{-1}$	γ	C, $(\mathrm{GeV}/c)^{-1}$	C_1	C_2, $(\mathrm{GeV}/c)^{-1}$	C_3, $(\mathrm{GeV}/c)^{-2}$
π^+	153	5.55	1		5.3667	−3.5	0.8334
π^-	127	5.3	3		7.0334	−4.5	1.667
K^+	8.85	4.05		2.5			
K^-	9.3	3.8		8.3			

$A + B \rightarrow C + D$, $p_{\mathrm{c.m.,max}}$ is given by (ref. 13 and appendix A)

$$p_{\mathrm{c.m.,max}} = \left[\frac{(s - m_C^2c^4 - M^2c^4)^2 - 4m_C^2M^2c^8}{4s} \right]^{1/2} \quad (10)$$

where

$$M^2 = (m_A + m_B)^2 \quad (11)$$

Once again, for the current problem, this simplifies to

$$p_{\mathrm{c.m.,max}} = \left[\frac{(s - m_\pi^2c^4 - 4m_p^2c^4)^2 - 16m_\pi^2m_p^2c^8}{4s} \right]^{1/2} \quad (12)$$

Hence, this allows the Lorentz-invariant cross sections from equations (4) and (6) to be expressed solely in terms of the masses, momenta, and energies of the particles (and the parameters given in table 1).

Spectral Distribution

The spectral distribution $d\sigma/dE$ is expressed in terms of the invariant cross section via (ref. 11 and appendix A)

$$\frac{d\sigma}{dE} = \frac{\pi}{p_{//}} \int \frac{E\, d^3\sigma}{d^3p}\, d(p_\perp^2) \quad (13)$$

By inserting the invariant cross sections from equations (4) and (6) into equation (13) and integrating, a representation of $d\sigma/dE$ will be produced in terms of proton kinetic energy (T_{lab}) and meson parallel momentum ($p_{//}$). Because of the nonlinear exponential form of equations (4) and (6) and the nonlinear integration in equation (13), the integrals cannot be performed analytically. A numerical procedure utilizing adaptive Gaussian 32-point quadrature was implemented (see the computer program in appendix B); this procedure produced the data points seen in figure 1, which shows the spectral distribution for both positively and negatively charged pions and kaons at a variety of proton energies.

By fitting a function of $p_{//}$ and T_{lab} to the data in figure 1, the parameterization will be solved. Clearly, the data are well represented by decreasing the exponentials, and the solid-line curves in the figures represent the best fits obtained by utilizing the following exponential function to represent $d\sigma/dE$:

$$\frac{d\sigma}{dE} = \frac{\pi}{p_{//}} \alpha \exp(-\beta p_{//}^{\chi}) \quad (14)$$

where α, β, and χ vary with respect to the incident proton kinetic energy (T_{lab}) and must be parameterized as such. This procedure is accomplished by determining which values of α, β, and χ best match the various proton energies from equation (14) with the output from equation (13), and then by finding functions that accurately provide the same values if given simply the proton kinetic energy (T_{lab}). The following functions were found to be extremely accurate:

For α,

$$\alpha = \alpha_1 \ln(\alpha_2 + T_{\mathrm{lab}}) + \alpha_3 \ln(\alpha_4 + T_{\mathrm{lab}})^2 + \alpha_5 \quad (15)$$

for β,

$$\beta = \beta_1 T_{\mathrm{lab}}^{-\beta_2} \ln(\beta_3 + T_{\mathrm{lab}})^{-\beta_4} - \beta_5 T_{\mathrm{lab}}^{-\beta_6} \quad (16)$$

and for χ,

$$\chi = \chi_1 \frac{T_{\mathrm{lab}}^{\chi_2} + \chi_3}{T_{\mathrm{lab}}^{\chi_4} + \chi_5} \quad (17)$$

where the subscripted parameters of equations (15), (16), and (17) are given in table 2. The accuracy of

Table 2. Parameters for Representations of α, β, and χ Used in Equations (15), (16), and (17)

Parameter		π^+	π^-	K^+	K^-
α	α_1	1.6209	2.0555	0.513	1.0023
	α_2	−.0992	2.5949	3.1993	15.612
	α_3	−.0958	−.546	−.0421	−.0711
	α_4	−.0217	689.37	−.8582	2.1746
	α_5	2.5884	21.24	−.6736	−2.7214
β	β_1	57.4728	72.1723	25.4335	66.5962
	β_2	−.6054	−.6595	−.5313	−.5619
	β_3	.7049	.7049	−.1231	−.122
	β_4	−.534	−.5335	−.9639	−.9198
	β_5	42.169	51.658	12.887	28.343
	β_6	−.7979	−.8771	−.8233	−.8742
χ	χ_1	1.9096	1.8773	2.3452	33.621
	χ_2	1.6939	1.4223	1.1829	−.0301
	χ_3	.5613	.4237	.5525	−.5638
	χ_4	1.7668	1.4898	1.2645	−.5344
	χ_5	.6666	.5082	.7275	6.361

equation (14) is evidenced by the very fine agreement between the data points and the solid-line curves seen in figure 1.

Results

The agreement between the fitted approximation using equation (14) and the output from the numerical quadrature used to evaluate the integral of equation (13) is found to be extremely accurate for a wide range of proton energies relevant to cosmic-ray interactions.

A final interesting point in regard to Nagamiya and Gyulassy's representation (ref. 13) of the maximum transferable momentum (eqs. (10) and (12)) is that the representation involves a square root that allows a lower bound to be set on T_{lab}, below which the root becomes imaginary. Because the only variables besides T_{lab} that are involved in the square root are the masses of the particles (ref. 14), the following lower limits on T_{lab} are obtained for meson production in proton-proton collisions. Thus, for pions,

$$T_{lab} > 0.29 \text{ GeV}$$

and for kaons,

$$T_{lab} > 1.12 \text{ GeV}$$

which are just the threshold energies.

Concluding Remarks

A parameterization of the energy-differential cross sections for charged pion and kaon production from proton-proton collisions has been derived that accurately reproduces results obtained by numerically integrating the Lorentz-invariant cross section. Because the parameterization is simply a function of incident proton kinetic energy and outgoing meson parallel momentum, the results can be easily reproduced and should be very useful in computer transport codes that have not yet been tested with meson cross sections.

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March 30, 1995

Appendix A

Proofs of Formulas Used in Text

Mandelstam Energy Variable

The Mandelstam energy variable (s) for the reaction

$$A + B \rightarrow C + D$$

by definition, is given as

$$s \equiv (p_A + p_B)^2 / c^2$$

In lab frame, $\boldsymbol{p}_B = 0$ and $E_B = m_B c^2$. Thus, s is expressed as

$$\begin{aligned} s &\equiv (p_A + p_B)^2 / c^2 \\ &= (p_A^2 + p_B^2 + 2 p_A p_B) / c^2 \\ &= (m_A^2 c^2 + m_B^2 c^2 + 2 p_A p_B) / c^2 \\ &= m_A^2 + m_B^2 + (2/c^2)\, p_A p_B \\ &= m_A^2 + m_B^2 + (2/c^2)\, (E_A/c, p_A) \cdot (E_B/c, p_B) \\ &= m_A^2 + m_B^2 + (2/c^2)\, (E_A/c, p_A) \cdot (m_B/c, 0) \\ &= m_A^2 + m_B^2 + (2 E_A m_B / c^2) \end{aligned}$$

Maximum Transferable Momentum

The maximum transferable momentum for the reaction

$$A + B \rightarrow C + D$$

in the center-of-mass frame where $\boldsymbol{p} \equiv \boldsymbol{p}_A = -\boldsymbol{p}_B$ and $\boldsymbol{q} \equiv \boldsymbol{p}_C = -\boldsymbol{p}_D$, is given by

$$q_{\max} = [\, (s - m_C^2 c^4 - M^2 c^4)^2 - 4 m_C^2 M^2 c^8 \,]^{1/2} / 2cs^{1/2}$$

where

$$M = m_A + m_B$$

Here, s is defined as

$$\begin{aligned} s \equiv (p_A + p_B)^2 / c^2 &= (p_C + p_D)^2 / c^2 \\ &= [\, (E_C/c, \boldsymbol{q}) + (E_D/c, -\boldsymbol{q}) \,]^2 / c^2 \\ &= [\, (E_C/c) + (E_D/c, \boldsymbol{0}) \,]^2 / c^2 \\ &= (E_C + E_D)^2 \end{aligned}$$

which yields

$$E_C + E_D = s^{1/2} \tag{A1}$$

When relating energy and momentum via

$$E_C^2 = \boldsymbol{q}^2 c^2 + m_C^2 c^4$$

$$E_D^2 = (-\boldsymbol{q})^2 c^2 + m_D^2 c^4$$

the difference between E_C^2 and E_D^2 is given as

$$E_C^2 - E_D^2 = (m_C^2 - m_D^2)\, c^4$$

Now, since

$$E_C^2 - E_D^2 = (E_C + E_D)\, (E_C - E_D)$$

we have

$$E_C - E_D = (m_C^2 - m_D^2)\, c^4 / (E_C + E_D)$$

where $(E_C + E_D)$ is given by equation (A1). Hence,

$$E_C - E_D = (m_C^2 - m_D^2)\, c^4 / s^{1/2} \tag{A2}$$

Spectral Distribution

The spectral distribution is given as

$$\frac{d\sigma}{dE} = \frac{\pi}{c^2 p_{//}} \int \frac{E\, d^3\sigma}{d^3 p}\, d(p_\perp^2)$$

For proof, we let

$$\frac{E\, d^3\sigma}{d^3 p} = \frac{E\, d^3\sigma}{dp_{//}\, p_\perp\, dp_\perp\, d\theta} \tag{A3}$$

Integrating with respect to θ gives

$$2\pi\, \frac{E\, d^3\sigma}{d^3 p} = \frac{E\, d^2\sigma}{dp_{//}\, p_\perp\, dp_\perp} \tag{A4}$$

We can simplify further via

$$\begin{aligned} \boldsymbol{p}^2 &= p_\perp^2 + p_{//}^2 \\ E &= \left[(p_\perp^2 + p_{//}^2)\, c^2 + m_0^2 c^4 \right]^{1/2} \\ \frac{dE}{dp_{//}} &= \frac{p_{//} c^2}{E} \\ dp_{//} &= \frac{E\, dE}{p_{//} c^2} \end{aligned} \tag{A5}$$

and

$$d(p_\perp^2) = 2p_\perp\, dp_\perp$$
$$p_\perp dp_\perp = d(p_\perp^2)/2 \qquad \text{(A6)}$$

Substituting equations (A5) and (A6) into the denominator on the left-hand side of equation (A4) gives

$$2\pi \frac{E\, d^3\sigma}{d^3p} = \frac{E\, d^2\sigma}{\frac{E\, dE}{p_{//}c^2}\left[d(p_\perp^2)/2\right]}$$

and

$$\frac{\pi}{c^2 p_{//}} \frac{E\, d^3\sigma}{d^3p} = \frac{d^2\sigma}{dE\, d}(p_\perp^2) \qquad \text{(A7)}$$

Finally, integrating with respect to $p_\perp^2$ yields the desired result

$$\frac{d\sigma}{dE} = \frac{\pi}{c^2 p_{//}} \int \frac{E\, d^3\sigma}{d^3p}\, d(p_\perp^2) \qquad \text{(A8)}$$

which allows us to isolate E_C, E_D, and $\boldsymbol{q}$ in terms of m_C, m_D, and s as

$$E_C = \left[s + (m_C^2 - m_D^2)\, c^4\right] 2s^{1/2}$$
$$E_D = \left[s + (m_C^2 - m_D^2)\, c^4\right] 2s^{1/2}$$
$$\boldsymbol{q}^2 = (E_C^2/c^2) - m_C^2 c^2 = \left(\left\{ \left[s + \left(m_C^2 - m_D^2 \right) c^4 \right]^2 s^{1/2} \right\}^2 /c^2 \right) - m_C^2 c^2$$
$$= \left\{ \left[s + \left(m_C^2 - m_D^2 \right) c^4 \right]^2 - 4s m_C^2 c^4 \right\} /4sc^2$$
$$= \left[s^2 + 2s\left(m_C^2 - m_D^2 \right) c^4 + \left(m_C^2 - m_D^2 \right) 2c^8 - 4m_C^2 c^4 \right] /4sc^2$$
$$= \left[s^2 - 2s\left(m_C^2 + m_D^2 \right) c^4 + \left(m_C^2 - m_D^2 \right)^2 c^8 \right] /4sc^2$$

Note that $\boldsymbol{q}$ is the transferred momentum and that for given particles A, B, and C and given incident energy E_{beam}, the only variable left in $\boldsymbol{q}$ is m_D, which can be no larger than $m_A + m_B$. Hence, the maximum transferable momentum is given as

$$\boldsymbol{q}_{\max} = \{ s^2 - 2sc^4 \left[m_C^2 + (m_A + m_B)^2 \right] + \left[m_C^2 - (m_A + m_B)^2 \right]^2 c^8 \}^{1/2} /2cs^{1/2}$$

and if we let $M = m_A + m_B$, the result is

$$\boldsymbol{q}_{\max} = \left[s^2 - 2sc^4 (m_C^2 + M^2) + (m_C^2 - M^2)^2 c^8 \right]^{1/2} /2cs^{1/2}$$
$$= \left[\left(s - m_C^2 c^4 - M^2 c^4 \right)^2 - 4m_C^2 M^2 c^8 \right]^{1/2} /2cs^{1/2}$$

which is the desired result.

Appendix B

Computer Program of Spectral Parameterizations

The following computer program utilizes the parameterization given by Badhwar, Golden, and Stephens, (ref. 11) to plot a graph of the spectral distribution. The integration is performed by using adaptive Gaussian 32-point quadrature to find the integral.

The program is written in FORTRAN-77, which does not support the many fonts found in this paper; hence, naming the variables can become quite tedious. A complete list of the meaning of all the variables found in the main procedure is given as follows:

A, B, C, C1, C2, C3	parameterization constants (A_1, A_2, C, C_1, C_2, and C_3, respectively) from table 1
dppar	dummy variable used to vary pparcm inside loop
dsdE	= $d\sigma/dE$, energy-differential cross section
e	total energy of incoming proton
fac	represents one part of equation (1) that does not vary with respect to $p_{//}$ or $p_{\perp}$
intgrl	value of actual definite integral
invcs	value of invariant cross section, returned by function of same name
mkaon	mass of kaon
mp	mass of proton
mpion	mass of pion
particle	determines which particle the program will investigate
pcmmax	maximum transferable momentum
ppercm	= $p_{\perp}$, perpendicular component of c.m. momentum
pparcm	= $p_{//}$, parallel component of c.m. momentum
q	= q, from equation (5)
R	= γ, parameterization constant from table 1
s	Mandelstam energy
Tlab	kinetic energy of incident proton
x	= 1 – xtilde, as seen in equations (3) and (4)
xparsq	= $x_{//}^{*2}$, from equation (3)
xtilde	scaling variable found in equation (3)

```
      IMPLICIT DOUBLE PRECISION (a-z)
      INTEGER particle
      COMMON mass,A,B,C,C1,C2,C3,R,q,s,particle,fac,xparsq

*     mass of proton in GeV/c**2
      mp  = .93827231
*     mass of pion in Gev/c**2
      mpion = .1395679
*     mass of kaon in GeV/c**2
      mkaon = .493646
      pi = 3.141592653589793238

      do 2000 Tlab = 10.
      do 1000 particle = 1,4,1

      IF (particle.EQ.1) THEN
*         for positive pion...
          A  = 153.
          B  = 5.55
          R  = 1.
          C1 = 5.3667
          C2 = -3.5
          C3 = 0.8334
          mass = mpion
          open(50,file='intpp0010.dat')
      ELSEIF (particle.eq.2) then
*         for negative pion...
          A  = 127.
          B  = 5.3
          R  = 3.
          C1 = 7.0334
          C2 = -4.5
          C3 = 1.667
          mass = mpion
          open(50,file='intnp0010.dat')
      ELSEIF (particle.eq.3) then
*         for positive kaon...
          A = 8.85
          B = 4.05
          C = 2.5
          mass = mkaon
          open(50,file='intpk0010.dat')
      ELSEIF (particle.eq.4) THEN
*         for negative kaon...
          A = 9.3
          B = 3.8
          C = 8.3
          mass = mkaon
          open(50,file='intnk0010.dat')
```

```
      ENDIF

*     Mandelstam energy
      e = Tlab + mp
      s = 2.*(mp**2) + 2.*e*mp
*     maximum transferable momentum
      pcmmax = DSQRT(((s - mass**2 - 4.*(mp**2))**2 - 16.*(mass**2)*(mp**2))/(4.*s))
      write(6,*)particle,Tlab,e,s,' pcmmax = ',pcmmax

*     (neither perpendicular nor parallel component can be greater than pcmmax.)
*     (lower limit on integral is 0.  Upper limit is pcmmax**2, since we're
*     integrating with respect to the square of the perpendicular momentum,
*     and changed variables)
*     'fac' is a COMMON variable, and is used in invcs:
      fac  = 1. + 4.*(mp**2)/s

      dppar = pcmmax/1000.
      pparcm = 0.
      do 10 pparcm=0.,pcmmax,dppar
          xparsq = (pparcm/pcmmax)**2

          intgrl = AdaptiveGauss(DFLOAT(0),pcmmax**2)

          IF (intgrl.GE.0.) THEN
              dsdE = (pi/pparcm)*intgrl
              write(50,*)pparcm,dsdE
          ELSE
              write(6,*)'did not work: ',pparcm,intgrl
          ENDIF
10    continue

      close(50)

1000  continue
2000  continue

      stop
      end


*=======================================================================*
*     Adaptive Gaussian Quadrature...
*
*     sent in: lower and upper integration limits (ppercm limits)
*              returns the integral approximation 'Approx' of integral
*              of Lorentz invariant cross section (invcs).
*
      DOUBLE PRECISION FUNCTION AdaptiveGauss(Q1lim,Qulim)
      IMPLICIT DOUBLE PRECISION (a-z)
```

```
      INTEGER z,particle
      COMMON mass,A,B,C,C1,C2,C3,R,q,s,particle,fac,xparsq

      DIMENSION low(1000)
      DIMENSION mid(1000)
      DIMENSION upp(1000)
      DIMENSION sum(1000)
      DIMENSION tol(1000)
      DIMENSION sav(5)

      Approx = 0.
      z      = 1
      tol(z) = 1./(10.**10)
      low(z) = llim
      mid(z) = (llim + ulim)/2.
      upp(z) = ulim
      sum(z) = Gauss32(low(z),upp(z))

100   IF (z.GT.0) THEN
          s1 = Gauss32(low(z),mid(z))
          s2 = Gauss32(mid(z),upp(z))

          sav(1) = low(z)
          sav(2) = mid(z)
          sav(3) = upp(z)
          sav(4) = tol(z)
          sav(5) = sum(z)

          z = z - 1

          IF (DABS(s1 + s2 - sav(5)).LT.sav(4)) THEN
              Approx = Approx + (s1 + s2)
          ELSE
              IF (z.GE.999) THEN
                  AdaptiveGauss = -1.
                  goto 9999
              ELSE
*                 calculate for right-hand side subinterval:
                  z = z + 1
                  low(z) = sav(2)
                  upp(z) = sav(3)
                  mid(z) = (low(z) + upp(z))/2.
                  tol(z) = sav(4)/2.
                  sum(z) = Gauss32(low(z),upp(z))
*                 calculate for left-hand side subinterval:
                  z = z + 1
                  low(z) = sav(1)
                  upp(z) = sav(2)
                  mid(z) = (low(z) + upp(z))/2.
```

```
                tol(z) = tol(z-1)
                sum(z) = Gauss32(low(z),upp(z))
            ENDIF
        ENDIF
      GOTO 100
      ENDIF
      AdaptiveGauss = Approx

 9999 return
      end

*-----------------------------------------------------------------------------*
*     32-point Gaussian Quadrature, used in conjunction w/ AdaptiveQuad above
*
      DOUBLE PRECISION FUNCTION Gauss32(llim,ulim)
      Implicit Double Precision (a-z)
      Integer i,degree
      Dimension Wgt(32), Zero(32)
      degree = 32
      CALL LEGEND(Wgt,Zero)
      sum    = 0.0
      DO 300 i=1,degree
          x      = (Zero(i)*(ulim - llim) + llim + ulim)/2.
          NewWgt = Wgt(i)*(ulim - llim)/2.
          sum = sum + invcs(x)*NewWgt
300   CONTINUE
      Gauss32 = sum
      RETURN
      END

*-----------------------------------------------------------------------------*
*     Lorentz-invariant cross section of DSQRT(temp)....
*
      DOUBLE PRECISION FUNCTION invcs(temp)
          IMPLICIT DOUBLE PRECISION (a-z)
          INTEGER particle
          COMMON mass,A,B,C,C1,C2,C3,R,q,s,particle,fac,xparsq
          ppercm = DSQRT(temp)

          xtilde = DSQRT(xparsq + 4.*(ppercm**2 + mass**2)/s)
          x = 1. - xtilde
          if (x.GE.0.) then
              if (particle.LT.3) then
                  q     = (C1 + C2*ppercm + C3*(ppercm**2))/DSQRT(fac)
                  invcs = (A/(fac**R))*(x**q)*DEXP(-B*ppercm/fac)
              else
                  invcs = A*(x**C)*DEXP(-B*ppercm)
              endif
          else
```

```
            invcs = 0.
         endif
      return
      end

*=======================================================================*
*     Legend: Set initial values for Laguerre Weights and Points....
*
      SUBROUTINE LEGEND(WL,P)
            Implicit Double Precision (A-H,M,O-Z)
            Dimension WL(96),P(96)
*           Laguerre weights are WL,  Laguerre points are P
      WL(   1)=  0.70186100094700121189081332140 3D-02
      WL(   2)=  0.16274394730905660834407999715 3D-01
      WL(   3)=  0.25392065309262071368806568116 8D-01
      WL(   4)=  0.34273862913021416708170274745 3D-01
      WL(   5)=  0.42835898022226642258003526109 2D-01
      WL(   6)=  0.50998059262376232818325672013 8D-01
      WL(   7)=  0.58684093478535567615927970308 6D-01
      WL(   8)=  0.65822222776361790502752668885 5D-01
      WL(   9)=  0.72345794108848547956935881586 6D-01
      WL(  10)=  0.78193895787070243422189896875 8D-01
      WL(  11)=  0.83311924226946767677026706167 0D-01
      WL(  12)=  0.87652093004403934181634205913 3D-01
      WL(  13)=  0.91173878695763897750392601437 9D-01
      WL(  14)=  0.93844399080804524748700146119 5D-01
      WL(  15)=  0.95638720079274900862098540699 2D-01
      WL(  16)=  0.96540088514727850216301519026 3D-01
      WL(  17)=  0.96540088514727810317661571559 7D-01
      WL(  18)=  0.95638720079274919944056776444 1D-01
      WL(  19)=  0.93844399080804642709896512542 4D-01
      WL(  20)=  0.91173878695763932444862120974 0D-01
      WL(  21)=  0.87652093004403899487164686377 1D-01
      WL(  22)=  0.83311924226946741656174566514 9D-01
      WL(  23)=  0.78193895787070318015299363878 5D-01
      WL(  24)=  0.72345794108848428261016039186 9D-01
      WL(  25)=  0.65822222776361917137566415192 4D-01
      WL(  26)=  0.58684093478535610984014869728 8D-01
      WL(  27)=  0.50998059262376157357854467022 7D-01
      WL(  28)=  0.42835898022226735065709490868 4D-01
      WL(  29)=  0.34273862913021453137363270258 2D-01
      WL(  30)=  0.25392065309262001112505791056 2D-01
      WL(  31)=  0.16274394730905591445468960643 0D-01
      WL(  32)=  0.70186100094701142507527813485 5D-02
      P(   1)= -0.99726386184948166702124439098 0D+00
      P(   2)= -0.98561151154526842332881386710 1D+00
      P(   3)= -0.96476225558750652866457642176 1D+00
      P(   4)= -0.93490607593773976391471336455 7D+00
      P(   5)= -0.89632115576605218831840460325 1D+00
```

```
P(   6)= -0.84936761373257001206393823622 4D+00
P(   7)= -0.79448379596794244106305882269 2D+00
P(   8)= -0.73218211874028971131167509156 5D+00
P(   9)= -0.66304426693021524530369248395 8D+00
P(  10)= -0.58771575724076242919213797222 3D+00
P(  11)= -0.50689990893222941492624045167 8D+00
P(  12)= -0.42135127613063539514515198902 7D+00
P(  13)= -0.33186860228212766710242931367 2D+00
P(  14)= -0.23928736225213704039149753555 2D+00
P(  15)= -0.14447196158279649452760295957 9D+00
R(  16)= -0.48307665687738302572051818373 2D-01
P(  17)=  0.48307665687738307776222246303 6D-01
P(  18)=  0.14447196158279648064981515176 4D+00
P(  19)=  0.23928736225213704386094448750 6D+00
P(  20)=  0.33186860228212764628574760195 1D+00
P(  21)=  0.42135127613063536045068246949 1D+00
P(  22)=  0.50689990893222945655960387512 1D+00
P(  23)=  0.58771575724076234592541112533 6D+00
P(  24)=  0.66304426693021523142590467614 4D+00
P(  25)=  0.73218211874028975294503851500 8D+00
P(  26)=  0.79448379596794242718527101487 8D+00
P(  27)=  0.84936761373257001206393823622 4D+00
P(  28)=  0.89632115576605214668504117980 8D+00
P(  29)=  0.93490607593773976391471336455 7D+00
P(  30)=  0.96476225558750645927563738268 9D+00
P(  31)=  0.98561151154526836781766263584 4D+00
P(  32)=  0.99726386184948161151009315972 3D+00
 return
 end
```

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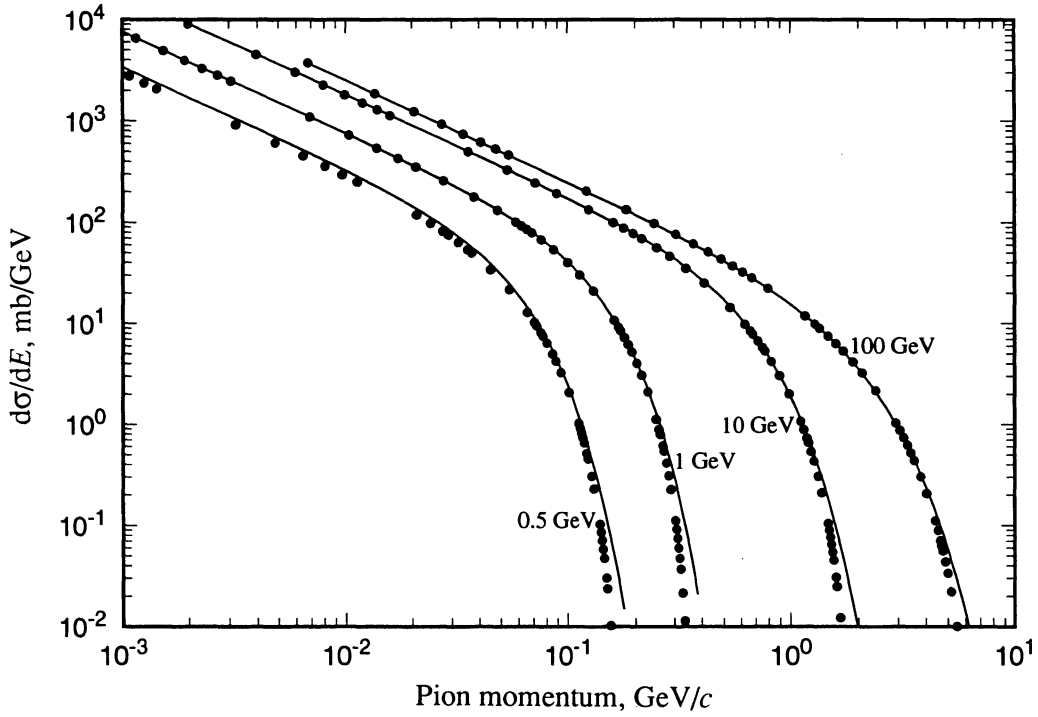
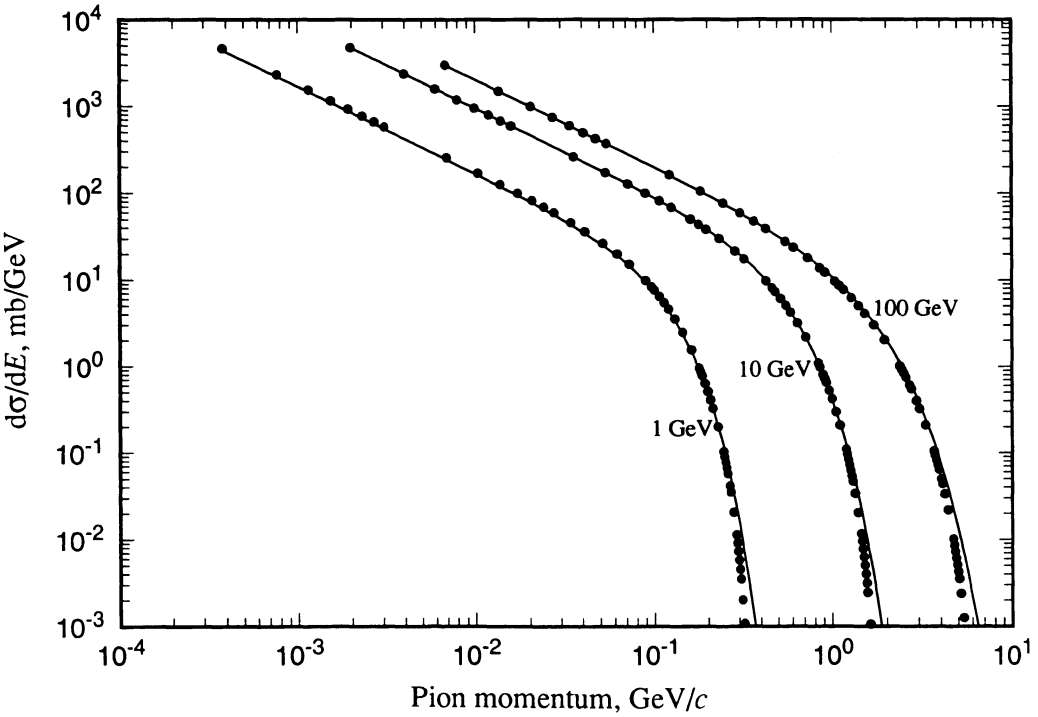
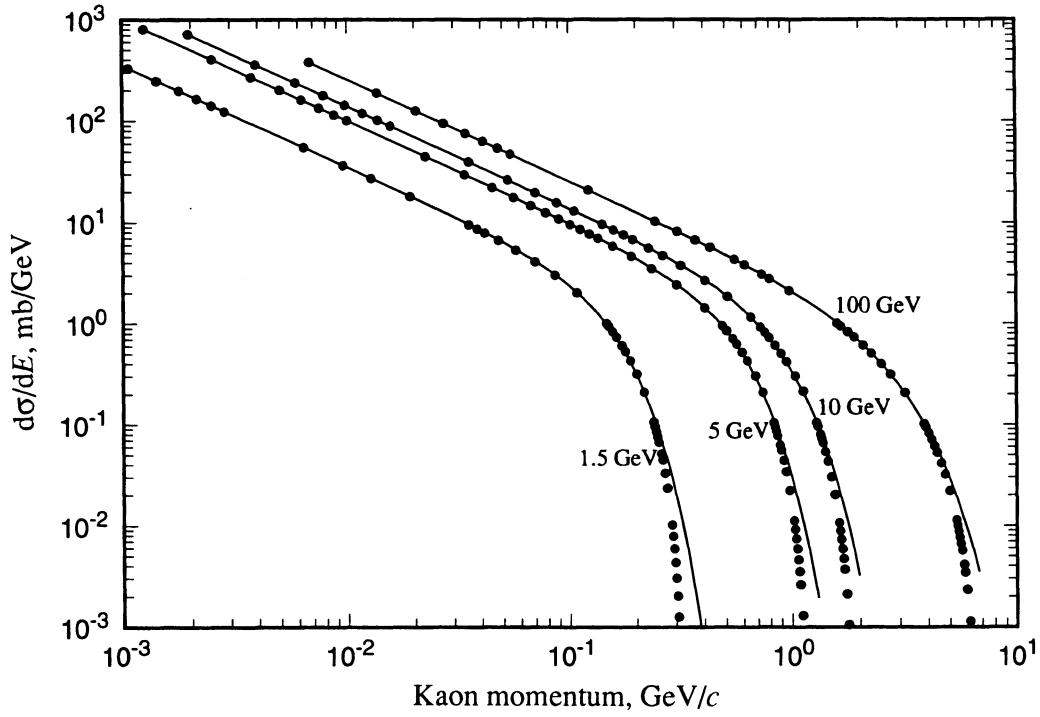


(a) Positive pion production.

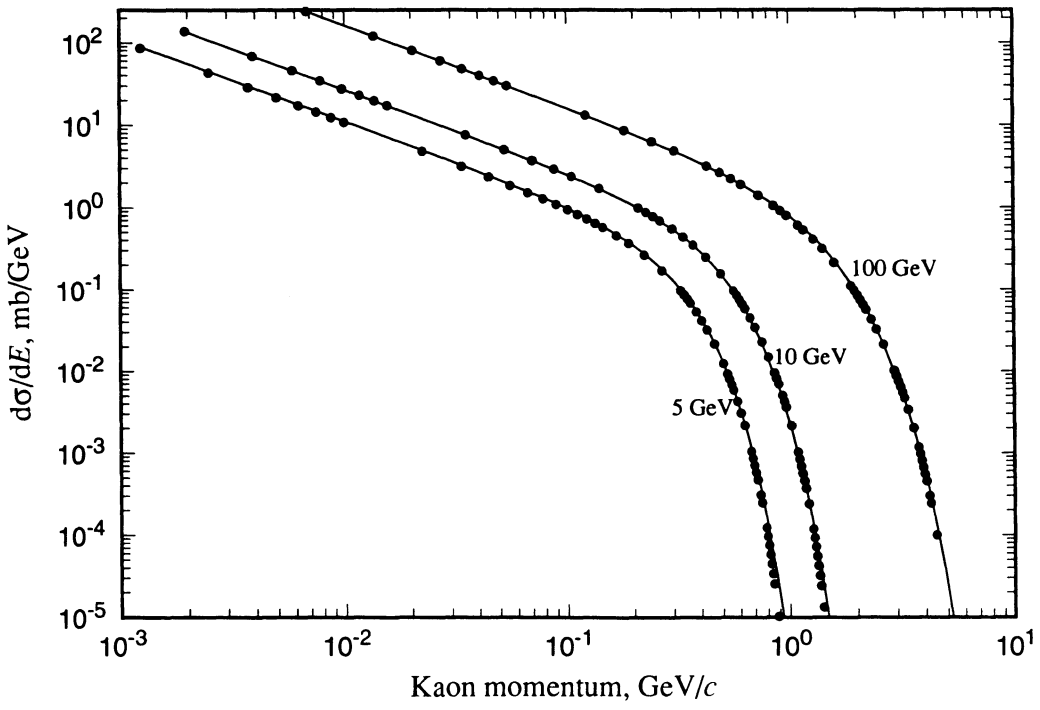


(b) Negative pion production.

Figure 1. Spectral distribution $d\sigma/dE$ plotted against parallel component of outgoing meson momentum. Data points denote actual values obtained by performing numerical quadrature on integral portion of equation (13); solid-line curves denote the fit approximation of equation (14).



(c) Positive kaon production.



(d) Negative kaon production.

Figure 1. Concluded.

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