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Abstract

The conversion of particle-energy spectra into a linear energy transfer (LET) distribution is a guide in assessing biologically significant components. The mapping of LET to energy is triple valued and can be defined only on open subintervals. A well-defined numerical procedure is found to allow generation of LET spectra on the open subintervals that are integrable in spite of their singular nature.

Introduction

A customary assumption is that the biological response (or risk) to radiation exposure is related to the energy absorbed per unit mass within a macroscopic volume of the biological material. This implies a certain uniformity in the energy deposited (which is true for the radiation of low linear energy transfer (LET)) such as from X-ray, γ-ray, and β-ray exposures. As experience was gained from exposures with high-LET radiation, the nonuniformity of the energy deposit was clearly an important determinant of biological response, and LET-dependent relative biological effectiveness factors came under intense study and provided the basis for the LET-dependent quality factor for use in radiation protection (ref. 1). The microdosimeter concept was developed as an instrument to measure such LET fluctuations.

Although the concept of an LET-dependent relative biological effectiveness (RBE) was useful for charged particles of relatively low kinetic energy, concerns for the biological effects of the high charge and energy ions (HZE) in deep space result from the lateral spread of the energy deposit that is larger than a typical cell nucleus (ref. 2). In this case, LET is not sufficient to define an RBE, but the track width also plays an important role. This was the origin of the limited LET concepts (refs. 1 and 3) as related to the lateral spread of the energy deposit. The importance of such track-structure effects on biological response has been demonstrated for a number of biological systems (ref. 4).

Even though LET as a biological-response indicator is most applicable to low charge Z and low-energy particles, LET is considered as a rough indicator for HZE particles as well. For this reason, we are interested in the generation of differential LET distributions as a guide to identify biologically significant components.

Differential LET Spectra

In radiobiology, relating biological response to the LET of the radiation environment is a tradition. For example, RBE and the quality factor Q are generally taken as being related to LET. As a consequence, the concept of LET spectra has played a role in estimating biological response. Unfortunately, this concept is most useful if the flux ($\phi_L(L)\ dL$) of particles with LET (that is, L) between L and $L + dL$ is known. This is generally found by knowing the energy flux ($\phi_E(E)\ dE$) of particles with energy (E) between E and $E + dE$, where L is known as a function of E so that

$$\phi_L(L) = \left|\frac{dL}{dE}\right|^{-1} \phi_E(E) \tag{1}$$

where the prefactor on the right-hand side is the Jacobian between the E and L spaces. The difficulty with this approach is that $dL/dE = 0$ at the maxima and minima of the LET curve; $\phi_L(L)$ must be replaced by the sum over the various branch functions as

$$\phi_L(L) = \sum_B \left|\frac{dL}{dE}\right|_B^{-1} \phi_E(E_B) \tag{2}$$

where E_B is the energy of each branch associated with L. That is, for all values of E_B,

$$L \equiv L(E_B) \tag{3}$$

Clearly, $\phi_L(L)$ does not exist for every value of L but is defined on open intervals not containing values for which $dL/dE = 0$. Furthermore, $\phi_L(L)$ is unbound on the open subintervals over which it is defined, even though $\phi_L(L)$ is integrable over its domain. From the above arguments, enough challenges obviously exist in finding a representation for $\phi_L(L)$, especially as some data set to be used by others in specific applications. This problem is simplified since $L(E)$ has but one maximum and one minimum other than at zero energy. Furthermore, $L(E)$ has continuous second derivatives allowing a Taylor series expansion as

$$L(E) \approx L(E_B) + \frac{1}{2} L''(E_B)(E - E_B)^2 \tag{4}$$

in the neighborhood of the branch limits. However, one finds

$$\phi_L(L) \approx \phi_E(E_B)\left[\sqrt{2\left|L''_B(L-L_B)\right|}\right]^{-1} \tag{5}$$

in the neighborhood of the branch points, where the subscript B denotes evaluation at the branch limit.

We implement the above considerations as follows. The LET is defined over a numerical grid given by the sequence $\{E_i\}$. The maximum and minimum branch points are found at $dL/dE = 0$ and are noted by $E_{\max}$ and $E_{\min}$, respectively. The sequence $\{E_i\}_L$ is defined as those values of E_i less than $E_{\max}$, with the main sequence $\{E_i\}_m$ being defined by $E_{\max} < E_i < E_{\min}$ and the sequence $\{E_i\}_H$ defined by $E_{\min} < E_i$. The three branch functions are then represented by

$$\{\phi_{Li}\}_B = \left\{\left|\frac{dE}{dL}\right|_i \phi_{Ei}\right\} \qquad E_i \in \{E_i\}_B \tag{6}$$

where B denotes one of the three branches (that is, $B = L$, m, or H). Giving a table of values $\{(\phi_{Li}, L_i)\}_B$ in order to reconstruct an adequate representation of the function over each branch is not sufficient because ϕ_{Li} is unbound near the branch limits and an extrapolation into the neighborhood of the branch limit must be provided. If $\{E_i\}$ is sufficiently close, then the LET spectrum may be represented as

$$\phi_L(L) \approx \phi_{Li}\left(|L_i - L_B| / |L - L_B|\right)^{1/2} \tag{7}$$

where L_i is the nearest grid value to the branch limit L_B in the appropriate domain. Thus, the data set required to reconstruct the LET spectrum is the branch limit values of $E_{\max}$, $E_{\min}$, $L_{\max}$, and $L_{\min}$ and the sequences $\{E_i\}$, $\{L_i\}$, and $\{\phi_{Ei}\}$. The numerical values of the above parameters depend on the charge and mass of the particles of the field. Thus, $E_{\max}$, $E_{\min}$, $L_{\max}$, and $L_{\min}$ must be specified for each ion type in the radiation field.

Stopping Powers

The energy imparted to the medium by a passing charged particle is related to the stopping powers (the energy loss per unit distance traveled) and is known as the linear energy transfer (LET), where $L \equiv S(E)$. To calculate the spectrum of energy deposit, we must evaluate the LET spectrum given by equation (1). Clearly, a method of generating dS/dE as a function of E is required. As explained elsewhere (ref. 5), the stopping powers used are taken from the fitted curves for protons and alpha particles of Ziegler at low energy (ref. 6), and they are extrapolated to the Bethe formula at high energy. The stopping power of ions with a charge greater than 2 is scaled using the effective charge formalism and the alpha particle stopping power. The stopping power is stored in a data array for 15 ion charge values (1 to 92) over an energy grid of 60 points between 10 keV and 50 GeV. Extrapolation to lower energy and higher energy is accomplished by making the stopping power and its first derivative continuous at the boundaries of the energy grid. The numerical values are scaled by $\ln\left[\frac{1}{Z^2}S(\ln E, Z)\right]$ to minimize numerical errors.

The scaled array is fitted by a two-dimensional spline with appropriate boundary conditions from which the derivative of $S(E)$ with respect to E is found. The derivatives, which are shown in figure 1, vanish at the LET maximum at low energy and the LET minimum at high energy because of relativistic effects. The energies associated with the maximum are shown in figure 2 along with the corresponding energies at the minimum. Figure 3 shows the corresponding values of $L_{\max}$ and $L_{\min}$. These provide the basic data for constructing differential LET (that is, L) spectra.

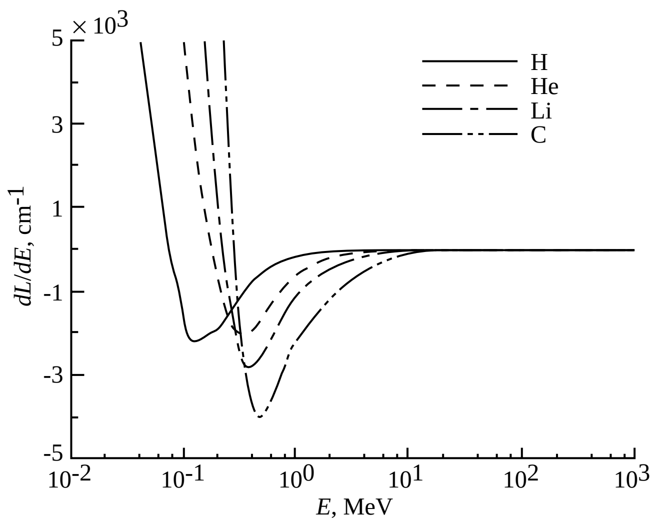

Figure 1. Derivatives of LET for several ions.

Equilibrium Spectra

Regardless of whether the source of ions is at the boundary or an interior volume source, the fluence approaches a characteristic equilibrium spectrum that depends on the composition of the medium. The equilibrium spectrum for volume sources has been

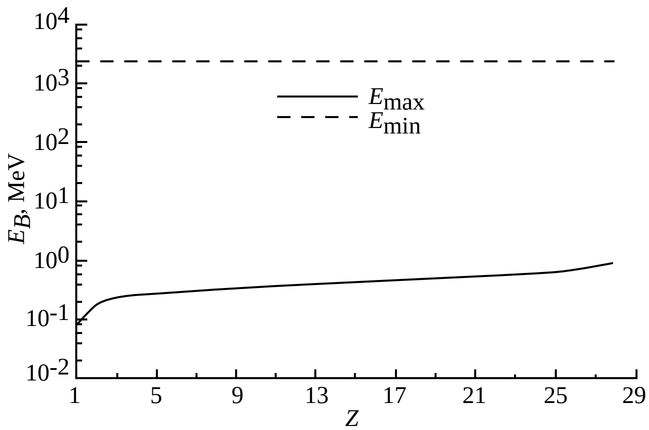

Figure 2. Branch limits as a function of atomic number.

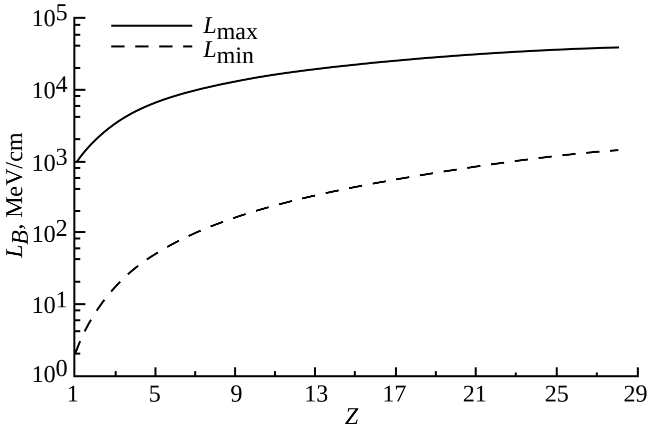

Figure 3. Values of LET at branch limits.

given elsewhere (ref. 7). The fluence within a medium at a distance x from the boundary is

$$\phi(x, E) = \frac{S(E_x)}{S(E)} \phi(0, E_x) \tag{8}$$

where $\phi(0, E)$ is the fluence at the boundary, $E_x = R^{-1}[R(E) + x]$ is the energy at the boundary, and $R(E)$ is the range of the particle. The $\phi(x, E)$ term of equation (8) results from (ref. 7)

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} S(E)\right] \phi(x, E) = \text{Nuclear gains and losses} \tag{9}$$

where the nuclear effects are assumed to be 0. We note that when $x \gg R(E)$, then $E_x \approx R^{-1}(x)$ and

$$\phi(x, E) \approx \frac{S\left[R^{-1}(x)\right] \;\; \phi\left[0, R^{-1}(x)\right]}{S(E)} \tag{10}$$

which is the equilibrium spectral limit. The LET spectra for $x = 0$, 0.5, 1, 3, and 5 cm of tissue are shown in figure 4 for three large solar events of solar cycles 19 and 20. The nuclear reaction products are neglected (refs. 7 and 8). One may observe in figure 4 that the equilibrium spectrum is achieved rather quickly above 40 MeV/cm corresponding to 12 MeV protons. Equilibrium is achieved above 10 MeV/cm for the 5-cm depth corresponding to 60 to 70 MeV protons. The limit of a pure equilibrium spectrum (for particle type j) at all energies is given by

$$\phi_E(E) = \frac{c}{S_j(E)} \tag{11}$$

where c is a constant. The equilibrium differential LET spectra for $c = 1$ are shown in figure 5.

Integral LET Spectra

The integral LET spectrum is given as

$$\Phi(> L) = \int_L^{L_{max}} \phi_L(L') \; dL' \tag{12}$$

which may be related to integral energy spectra as

$$\Phi(> L) = \Phi(> E_1) - \Phi(> E_2) + \Phi(> E_3) \tag{13}$$

where E_1, E_2, and E_3 are the three roots (branch functions) of

$$S(E) = L \tag{14}$$

The three branch functions of equation (13) are shown in figure 6 for a range of L values. The integral LET spectra for the three solar flares shown previously are shown in figure 7. To gain perspective, we also evaluate the integral equilibrium spectrum of equation (11) as

$$\Phi(> E) = \int_E^{\xi} \frac{dE'}{L(E')} = R(\xi) - R(E) \tag{15}$$

where we arbitrarily take the variable ξ to be 10 GeV. The integral LET spectrum for $\xi \geq E_3$ is then

$$\Phi(> L) = R(E_2) - R(E_1) + R(\xi) - R(E_3) \tag{16}$$

If we examine only the high-LET region, then

$$\Phi(> L) = R(E_2) - R(E_1) \tag{17}$$

Results from equation (17) are shown in figure 8. Thus, the integral LET spectra are characterized by the main branch of the LET curve as

$$\Phi(> L) \approx R(E_2) \tag{18}$$

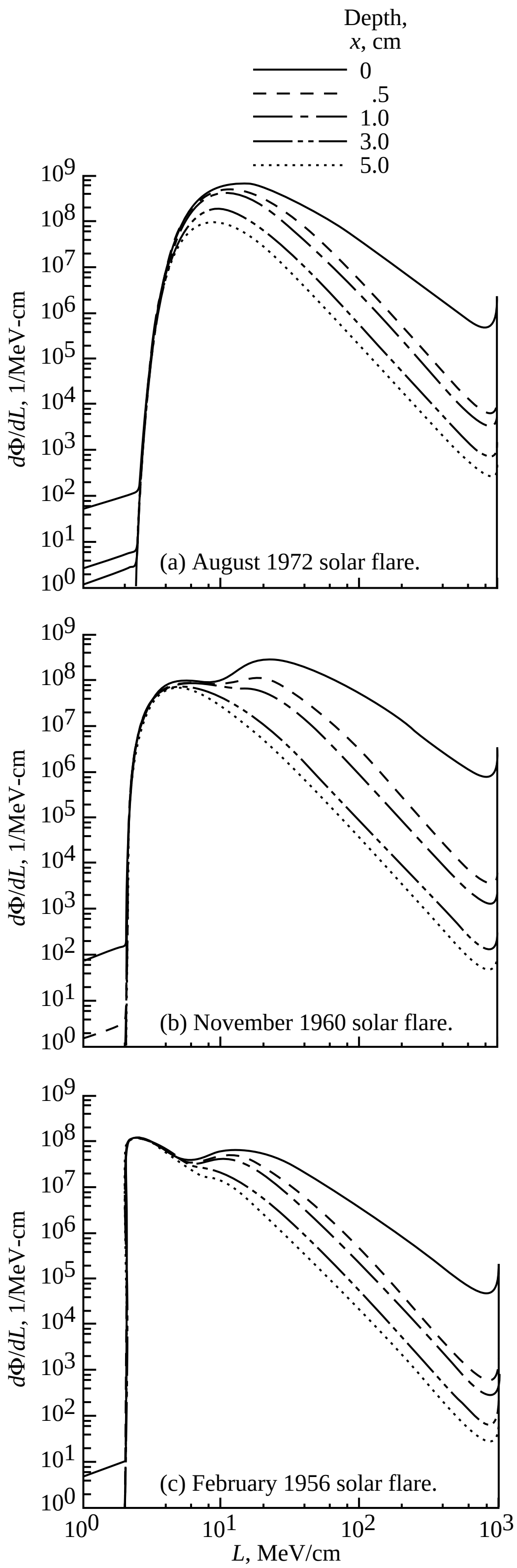

Figure 4. Differential LET spectra for three solar flares of cycles 19 and 20.

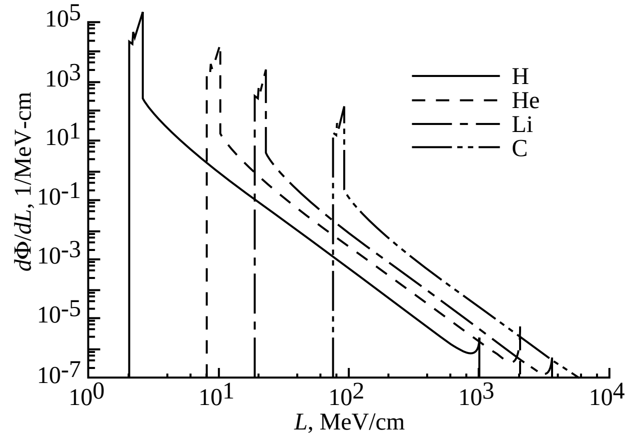

Figure 5. Equilibrium differential LET spectra for several ions.

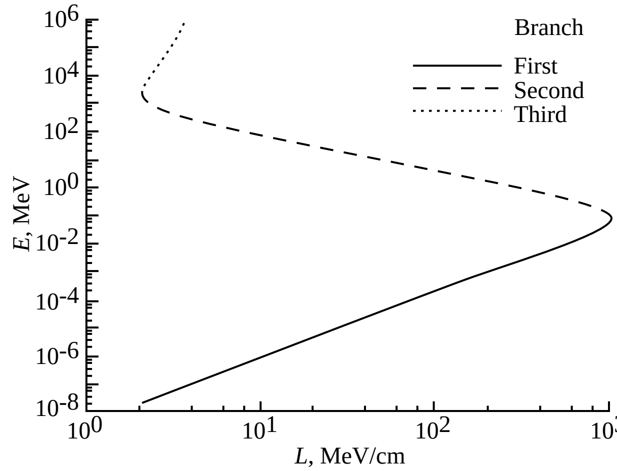

Figure 6. The three branch functions for protons.

except at the highest LET values where

$$\Phi\left(>L\right) \approx \left(E_2 - E_1\right) / S\left(E_{\max}\right) \qquad (19)$$

The integral LET spectra associated with the three solar events in figure 4 are shown in figure 7 and should be compared in shape to the equilibrium curve for protons in figure 8.

As an application of the present procedure, the differential LET spectra for the solar minimum environment are shown in figure 9 at three depths in tissue-equivalent material behind a slab of aluminum shielding of 5 g/cm^2. The reduction of high-LET components at larger tissue depths increases the low-LET component distributions as shown. The

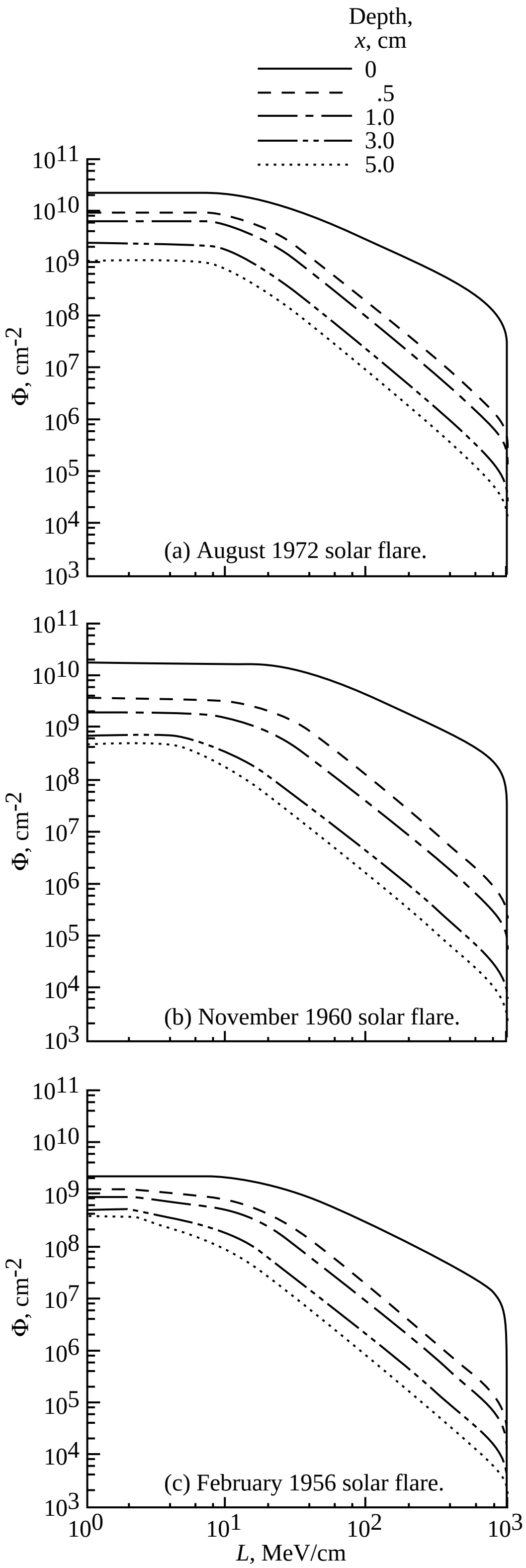

Figure 7. Integral LET spectra for three solar events of cycles 19 and 20.

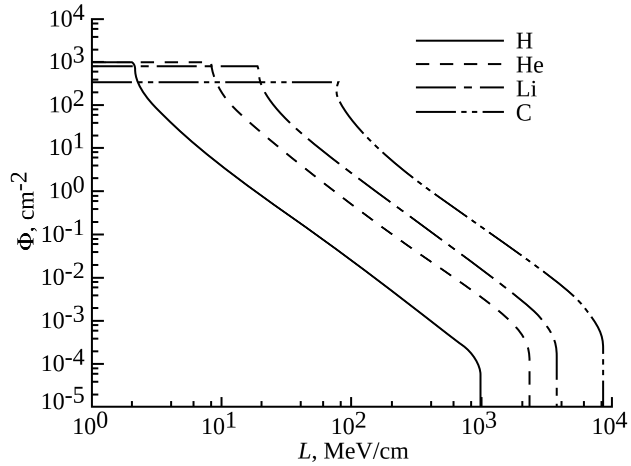

Figure 8. Equilibrium integral LET spectra for several ions.

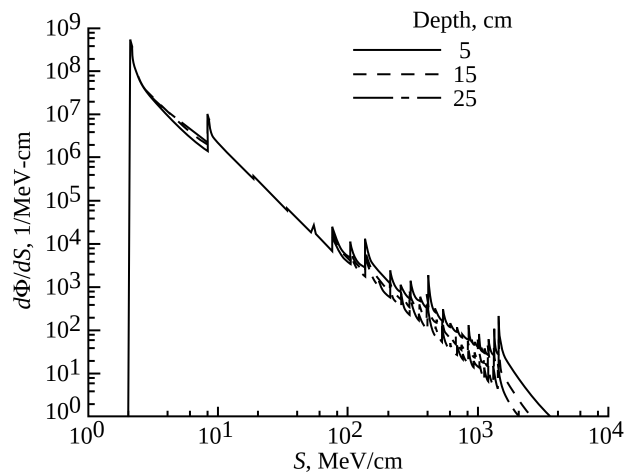

Figure 9. Differential LET spectra at solar minimum in tissue shielded by 5 g/cm^2 of aluminum.

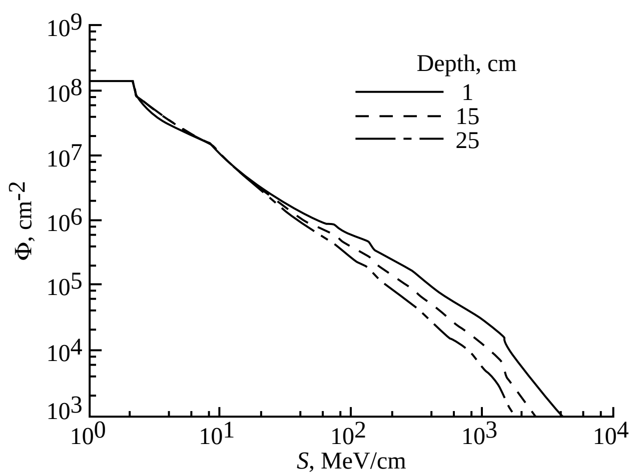

Figure 10. Integral LET spectra at solar minimum in tissue shielded by 5 g/cm^2 of aluminum.

corresponding integral LET spectra are shown in figure 10.

Concluding Remarks

The problem of generating linear energy transfer (LET) spectra has been clearly delineated. Although integral LET spectra are continuous functions, the differential LET spectra are defined only over open subintervals and are unbound near the open end points. An accurate method of constructing the spectra over their domain has been given. These methods will be useful in analyzing amplications of shielding for protection of biological and electronic equipment.

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October 19, 1992

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